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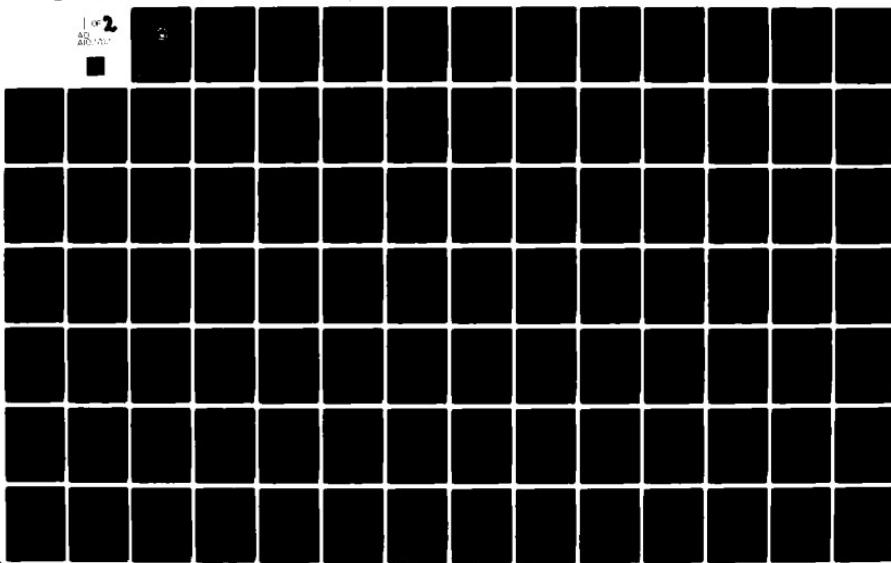
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AN ADAPTIVE KALMAN IDENTIFIER AND ITS APPLICATION TO LINEAR AND ETC(U)  
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**NAVAL POSTGRADUATE SCHOOL**  
Monterey, California



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**THESIS**

AN ADAPTIVE KALMAN IDENTIFIER  
AND ITS APPLICATION TO  
LINEAR AND NON-LINEAR ARMA MODELING.

by

Leopoldo M. Mayoral

11 Mar 1981

Thesis Advisor:

S.R. Parker

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#20 - ABSTRACT - (CONTINUED)

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It is shown that the Adaptive least mean square (LMS) Adaptive Recursive LMS and Adaptive Lattice filters are special suboptimal cases of the AKI. The convergence and modeling properties are compared with those of the AKI by simulation using various types of data.

With minor modification, the AKI algorithm was used to identify the linear and non-linear ARMA models of the phase locked loop (PLL). A discrete PLL using a forward Euler integration scheme was used as a source of non-linear data. The AKI technique appears to enable one to discern when a potential non-linear system enters into its non-linear mode of operation.

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An Adaptive Kalman Identifier  
and Its Application to  
Linear and Non-Linear ARMA Modeling

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## ABSTRACT

The problem of accurately replicating the parameters which define a given system for the purposes of implementing modern control strategies is important. Using an Autoregressive-Moving Average (ARMA) representation for the unknown system, a model is identified by processing input/output data to estimate the coefficients associated with the ARMA equation. Identification of unknown system parameters using Kalman filtering methods was accomplished by augmenting the state vector. In this thesis the Kalman filter is formulated so that parameters can be identified explicitly. We call this approach the Adaptive Kalman Identifier (AKI).

It is shown that the Adaptive least mean square (LMS), Adaptive Recursive LMS and Adaptive Lattice filters are special suboptimal cases of the AKI. The convergence and modeling properties are compared with those of the AKI by simulation using various types of data.

With minor modifications, the AKI algorithm was used to identify the linear and non-linear ARMA models of the phase locked loop (PLL). A discrete PLL using a forward Euler integration scheme was used as a source of non-linear data. The AKI technique appears to enable one to discern when a potential non-linear system enters into its non-linear mode of operation.

## TABLE OF CONTENTS

I.	INTRODUCTION -----	13
	A. BACKGROUND -----	13
	B. INVESTIGATIONS AND CONTRIBUTIONS -----	16
	C. ORGANIZATION -----	17
II.	FORMULATION OF THE PROBLEM -----	18
	A. DETERMINISTIC CASE -----	18
	B. NON DETERMINISTIC CASE -----	20
	1. Autoregressive Form ( $a_i = 0$ ) -----	24
	2. Moving Average Form ( $b_i = 0$ ) -----	27
	3. Autoregressive-Moving Average Form -----	30
	C. OBSERVATIONS -----	33
III.	COMPARISON BETWEEN THE ADAPTIVE MA KALMAN IDENTIFIER AND WIDROW LMS ADAPTIVE FILTER -----	36
	A. PRELIMINARIES -----	36
	B. THE COMPARISON -----	38
	C. OBSERVATIONS -----	41
IV.	COMPARISON BETWEEN THE ADAPTIVE KALMAN IDENTIFIER AND FEINTUCH ADAPTIVE RECURSIVE LMS FILTER -----	43
	A. PRELIMINARIES -----	43
	B. THE COMPARISON -----	47
	C. OBSERVATIONS -----	53
V.	SYNTHETIC DATA GENERATING PLANTS -----	54
	A. AUTOREGRESSIVE-MOVING AVERAGE DATA -----	54
	1. Moving Average Data (Case 1) -----	55
	2. Autoregressive Data (Case 2) -----	55
	3. Autoregressive-Moving Average Data (Case 3) -----	56

B.	PHASE LOCKED LOOP DATA -----	57
VI.	SOFTWARE -----	67
A.	ADAPTIVE KALMAN IDENTIFIER -----	67
B.	LMS ADAPTIVE FILTER -----	69
C.	ADAPTIVE RECURSIVE LMS FILTER -----	73
VII.	NON-LINEAR IDENTIFICATION -----	75
VIII.	FINDINGS, CONCLUSION -----	81
	A. ORDER OF THE UNKNOWN SYSTEM IS KNOWN -----	81
	1. AKI vs LMS Adaptive Filter -----	81
	2. AKI Applied to Autoregressive Data -----	91
	3. AKI Applied to ARMA Data -----	93
	4. AKI vs Adaptive Recursive LMS Filter -----	98
	5. AKI Applied to PLL Data (Linear Region) -	103
	6. AKI Applied to PLL Data (Non-linear Region) -----	103
	B. ORDER OF THE UNKNOWN SYSTEM IS NOT KNOWN (OVERMODELING) -----	109
	1. AKI vs LMS Adaptive Filter -----	109
	2. AKI Applied to AR and ARMA Data -----	114
	C. CONCLUSION -----	115
	D. TOPICS FOR FURTHER CONSIDERATION -----	118
APPENDIX A:	The Discrete Weiner Problem -----	120
APPENDIX B:	Program ARMA -----	124
APPENDIX C:	Program PLL -----	126
APPENDIX D:	Program ADAPTSN -----	131
APPENDIX E:	Program LMS -----	147
APPENDIX F:	Program LMSR -----	151

LIST OF REFERENCES -----	158
INITIAL DISTRIBUTION LIST -----	161

## LIST OF TABLES

### TABLE

6.1	AKI Flag Meanings -----	68
6.2	Valid Flag Combinations -----	69
7.1	Non-linear Weighting Coefficients Calculated Using Three Methods -----	80
8.1	Actual vs AKI Estimates for Coefficients of Equation 8.3 -----	93
8.2	Actual vs AKI Estimates for Coefficients of Equation 8.4 at the 371st Iteration -----	94
8.3	Actual vs AKI and Adaptive Recursive LMS Filter Estimates for Coefficients of Equation 8.6 -----	100
8.4	Actual vs AKI Estimates of the ARMA Repre- sentation for the PLL (Linear Region) -----	103
8.5	Overmodeling of a MA Process Using the AKI ( $k = 250$ ) -----	113
8.6	Overmodeling of a MA Process Using the AKI of orders AR = 2, MA = 6 -----	113

## LIST OF FIGURES

### FIGURE

2.1	Adaptive Kalman Identifier -----	34
3.1	LMS Adaptive Filter Implementation of System Identification -----	37
4.1	Adaptive Recursive LMS Filter Applied to System Identification -----	48
5.1	ARMA Digital Plant -----	56
5.2	PLL Block Diagram -----	58
5.3	Final PLL Block Diagram -----	59
5.4	PLL Step Response, Linear Region -----	60
5.5	PLL Step Response, Non-linear Region -----	61
5.6	Discrete PLL Block Diagram -----	62
5.7	Programmable PLL Implementing Forward Euler Integration -----	63
5.8	Discrete PLL Step Response, Linear Region -----	64
5.9	Discrete PLL Step Response, Non-linear Region -----	65
6.1	AKI General Program Flow -----	70
6.2	LMS Filter Program Flow -----	72
6.3	LMSR Filter Program Flow -----	74
7.1	PLL, Third Order Taylor Series Approximation -	76
8.1	LMS Adaptive Filter Weights, $k_s = -.200$ -----	83
8.2	LMS Adaptive Filter Weights, $k_s = -.300$ -----	84
8.3	LMS Adaptive Filter Weights, $k_s = -.400$ -----	85
8.4	LMS Adaptive Filter Weights, $k_s = -.500$ -----	86
8.5a	LMS Adaptive Filter Weights, $k_s = -.600$ -----	87

**FIGURE**

8.5b	LMS Adaptive Filter Weights, $k = -.200$ , with Increased Measurement Noise -----	88
8.6	AKI Moving Average Plant Coefficients -----	89
8.7	LMS Adaptive Filter Weights -----	89
8.8	AKI Averaged Gains for Moving Average Model --	90
8.9	AKI Moving Average Instantaneous Gains -----	90
8.10	AKI Autoregressive Plant Coefficients -----	92
8.11	AKI Autoregressive Averaged Gains -----	92
8.12	AKI(4,5) Computed Coefficients, Perry's Example -----	95
8.13	AKI(4,5) Average Gain Values -----	95
8.14a	Pole/zero Models Produced by AKI and Lattice Algorithms for a Plant with the Character- istic Transfer Function of Equation (8.4) ----	96
8.14b	Pole/zero Models Produced by AKI and Lattice Algorithms for a Plant with the Character- istic Transfer Function of Equation (8.4) at the 371st Iteration -----	97
8.15	Instantaneous Gain Values, AKI(4,5) Model ----	98
8.16	AKI(2,2) Computed Coefficients, Feintuch Example -----	101
8.17	LMSR Computed Coefficients, Feintuch Example -	101
8.18	AKI(2,2) Averaged Gains, Feintuch Example ----	102
8.19	Pole/zero Models Produced by AKI and Adaptive Recursive LMS Algorithms for a Plant with the Characteristic Transfer Function of Equation 8.6 -----	102
8.20	PLL, AKI(2,3) Computed Coefficients -----	104
8.21	Averaged Gain Values, AKI(2,3) PLL Model ----	104
8.22	PLL Root Locus Analysis vs Computed Roots of the AKI ARMA Model -----	106

**FIGURE**

8.23	Variation of $\bar{a}$ vs Input Noise Signal Power ---	108
8.24	Comparison of $\frac{\sin x}{x}$ and P.D.F. of Input Noise Signal -----	110
8.25a	AKI(0,4) Model of a Third Order MA Process ----	112
8.25b	Fourth Order LMS Filter Weights of a Third Order MA Process -----	112
8.26	AKI(3,1) Overmodeled AKI(4,2) -----	116
8.27	AKI(3,1) Overmodeled AKI(5,5) -----	116
8.28	AKI(4,5) Overmodeled AKI(5,6) -----	117
A.1	The Discrete Weiner Problem -----	121

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## I. INTRODUCTION

The accomplishments in the area of microprocessor technology in the last decade have made a noticeable impact in the area of modern control. The desire to implement modern control theories taking advantage of these advancements, has made it imperative that the engineer attempt to mathematically replicate the system he ultimately desires to control. Efforts in this regard have, hence, generated a growing interest in the area of system identification [Ref. 1,2,3,4]. By implication this thesis concerns itself with discrete/digital signal processing.

### A. BACKGROUND

System identification or modeling can be accomplished by innovative application of existing techniques which were generally considered filtering or state estimation methods [Ref. 5]. Previous research efforts have, for obvious reasons, focused on linear modeling; however, there is a rising interest in non-linear modeling methods [Ref. 6,7,8].

An attractive form for the representation of an unknown system is the Autoregressive-Moving Average (ARMA) equation,

$$y(k) = \sum_{i=0}^{\infty} a_i u(k-i) - \sum_{i=1}^{\infty} b_i y(k-i) \quad (1.1)$$

which states that the present output,  $y(k)$ , is a linear combination of past outputs,  $y(k-i)$ , and of past and present

inputs,  $u(k)$ . Its attractiveness lies in its linear character and easily implementable structure using microprocessor/computer algorithms. Its relationship to the state space representation of a linear system has already been established [Ref. 9,10] making it germane to consider that a system is identifiable by an equation of the form (1.1). The system identification problem thus entails identifying the coefficients  $a_i$  and  $b_i$ .

There are several methods for computing the coefficients,  $a_i$  and  $b_i$ , however, it is not the intent of this brief introduction to attempt to develop even the majority of them. Nevertheless, it is practical to present a referenced history of those methods encountered in this thesis.

Adaptive algorithms for the purpose of estimating the coefficients of (1.1) have always been of interest. Widrow, using a least means square error criterion and implementing the method of steepest descent, developed an adaptive algorithm which estimated the coefficients of the moving average process associated with equation (1.1) [Ref. 11]. That is, the moving average model,

$$y(k) = a'_0 u(k) + a'_1 u(k-1) + a'_2 u(k-2) + \dots \quad (1.2)$$

where,

$$a'_0 = a_0$$

$$a'_1 = (a_1 - a_0 b_1)$$

$$a'_2 = (a_2 - a_0 b_2) - b_1 (a_1 - a_0 b_1)$$

$$\cdot$$

$$\cdot$$

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was identified. The LMS theory has since been extended to include block LMS filtering methods [Ref. 12,13,14] using various search techniques [Ref. 15,4(Chapter 5)]. These methods have enjoyed much popularity in the area of Linear Prediction and digital speech processing [Ref. 16,17].

The shortcoming of representing equation (1.1) by its equivalent moving average model (1.2) lies in the practical aspect of its implementation. That is, the infinite series represented by equation (1.2) must be truncated at some point resulting in an approximation which may not adequately represent equation (1.1). Hence, efforts to adaptively estimate the coefficients for both the autoregressive ( $b_i$ ) and moving average ( $a_i$ ) processes continued [Ref. 18,19,20] with various degrees of success. Feintuch's Adaptive Regressive LMS procedure [Ref. 18] is still a controversial issue [Ref. 19,20].

From yet a different direction, Anderson and Moore suggested that the Kalman filtering algorithms can be used as a means to identify the  $a_i$  and  $b_i$  coefficients<sup>1</sup> of equation (1.1) [Ref. 21, pp. 50-52]. This thesis exploits this application.

<sup>1</sup>Anderson and Moore in fact formulate the technique to compute the coefficients  $a_i, b_i$   $i = 1, 2, \dots, p$  where  $p = n+m$ .

## B. INVESTIGATIONS AND CONTRIBUTIONS

This thesis formulates an adaptive application of the Kalman filter to identify the coefficients of the general ARMA equation (1.1). It is shown that the LMS adaptive filter [Ref. 11] and the Adaptive Recursive LMS filter [Ref. 18] are (1) special cases of the Adaptive ARMA Kalman identifier and (2) sub-optimal with respect to the underlying least means square (LMS) error criterion upon which the LMS adaptive filter, the adaptive recursive filter and the Kalman filter are based. It is also demonstrated that the adaptive Kalman identifier has excellent convergence properties.

By comparison, it is noted that the Kalman algorithm accounts for measurement noise where the LMS algorithms do not, a heretofore unapproached problem by LMS algorithms. The results indicate that the suggested modification by Griffiths [Ref. 22] of the convergence factor,  $k_s$ , for the LMS adaptive filter is justified.

The application of the Kalman filter algorithm is extended to identification of the coefficients associated with a special case of the generalized non-linear ARMA model [Ref. 8]. The results of the non-linear simulations suggest a technique for determining when a potential non-linear system enters its non-linear operating regime. Such a technique can prove valuable when on-line performance evaluation of a known non-linear system is required.

Lastly, the connection between the Kalman filter algorithm and the lattice filter algorithm [Ref. 6] is made. An example

which demonstrates and compares the performance of both algorithms is given.

### C. ORGANIZATION

Chapter II presents and exploits the Kalman filter equations emphasizing its connections with the Yule-Walker and the discrete Weiner-Hopf equations. The theory is further developed to investigate the general ARMA case. Chapter III pursues the theoretical comparisons between the discrete Weiner-Hopf equation upon which the adaptive LMS filter is based and the MA form of the Kalman filter. The theoretical comparison between the Adaptive Recursive LMS filter and the general ARMA form of the Kalman filter is made in Chapter IV. The software methods by which linear and non-linear synthetic data is generated are discussed in Chapter V. A short user's description of the data processing programs and the options provided is given in Chapter VI. A non-linear application for the identification of the parameters of a non-linear plant is presented in Chapter VII followed by a discussion and presentation of the results in Chapter VIII.

## II. FORMULATION OF THE PROBLEM

Identifying the coefficients of the Autoregressive-Moving Average (ARMA) equation,

$$y(k) + b_1 y(k-1) + \dots + b_n y(k-n) = a_0 u(k) + a_1 u(k-1) + \dots + a_m u(k-m) \quad (2.1a)$$

$$y(k) = \sum_{i=0}^m a_i u(k-i) - \sum_{i=1}^n b_i y(k-i) \quad (2.1b)$$

can be formulated as an adaptive Kalman identification problem. That is, instead of using the well known Kalman Filter equations [Ref. 23,24] to recursively estimate the states of a system, one can utilize the Kalman filter equations to adaptively estimate the coefficients of either an Autoregressive (AR), Moving Average (MA), or Autoregressive-Moving Average (ARMA) process by proper definition of the quantities involved. We call this the Adaptive Kalman Identifier for obvious reasons.

### A. THE DETERMINISTIC CASE

Consider for the moment that the  $a_i$  and  $b_i$  are constant. Then by collecting a sufficient number of measurements of the input  $u(k)$  and the output  $y(k)$ , one can readily solve a set of linear equations for the  $a_i$  and  $b_i$  coefficients. The "sufficient number" that is needed is  $n+m+1$  which define the

$n+m+1$  linear equations which solve the  $n+m+1$  unknown coefficients. The matrix equation to be solved takes the form:

$$\begin{bmatrix} y(0) \\ y(1) \\ \vdots \\ y(m) \\ y(m+1) \\ \vdots \\ y(m+n) \end{bmatrix} = \begin{bmatrix} u(0) & 0 & \dots & 0 & 0 & \dots & \dots & \dots & 0 \\ u(1) & u(0) & & & y(0) & & & & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & & & & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & & & & \vdots \\ u(m) & u(m-1) & u(0) & y(m-1) & y(m-2) & y(0) & 0 & a_0 \\ \hline u(m+1) & & u(1) & y(m) & y(m-1) & y(1) & 0 & a_1 \\ & & & & & & & \vdots \\ & & & & & & & a_m \\ u(m+n) & \dots & u(m) & y(n-1) & y(n-2) & y(0) & \vdots & b_1 \\ & & & & & & & b_2 \\ & & & & & & & \vdots \\ & & & & & & & b_n \end{bmatrix} \quad (2.2)$$

$$Y = T \begin{bmatrix} a \\ \hline \cdots \\ b \end{bmatrix} \quad (2.3)$$

It is interesting to note that the  $(n+m+1) \times (n+m+1)$  matrix,  $T$ , is block symmetric. The solution to (2.3) is readily apparent, namely,

$$\begin{bmatrix} a \\ \hline \cdots \\ b \end{bmatrix} = T^{-1} Y \quad (2.4)$$

where  $T^{-1}$  is the inverse matrix of  $T$ . Since we have assumed that the elements of  $T$  are perfect noiseless measurements, and that we somehow know a-priori the number of unknown

coefficients, then  $T$  is of full rank, namely  $\text{rank}(T) = n+m+1$ , and the inverse of  $T$  exists.

### B. THE NONDETERMINISTIC CASE

Rarely can the coefficients be modeled so ideally. A more prudent and realistic model admits that the ARMA equation coefficients are subject to random perturbations. Further, it can be said that in general measurement devices introduce noise into the measurement data. Hence, the measurement model should reflect this fact. Developing the Adaptive Kalman Identifier along the same lines as [Ref. 21] we let,

$$a_i(k+1) = a_i(k) + w_i(k) \quad i = 0, 1, 2, \dots, m \quad (2.5a)$$

$$b_j(k+1) = b_j(k) + w_j(k) \quad j = 1, 2, \dots, n \quad (2.5b)$$

where the  $\{w_i(k), w_j(k)\}$  are samples from a zero mean, white, gaussian random process. Additionally, we assume that the noise sources are uncorrelated,

$$E[w_{a_r}(k)w_{a_s}(k)] = 0 \quad \text{for } r \neq s \quad (2.6a)$$

$$E[w_{b_r}(k)w_{b_s}(k)] = 0 \quad \text{for } r \neq s \quad (2.6b)$$

$$E[w_{a_r}(k)w_{b_s}(k)] = 0 \quad \text{for all } r, s \quad (2.6c)$$

Similarly, allowing for noisy measurement devices, the measurement equation (2.1b), is modelled as,

$$y(k) = \sum_{i=0}^m a_i u(k-i) - \sum_{i=1}^n b_i y(k-i) + v(k) \quad (2.7)$$

where the  $\{v(k)\}$  are samples from a zero mean, white, gaussian random process. It is also assumed that,

$$E[w_i(k) v(k)] = 0 \text{ for all } i \quad (2.8a)$$

$$E[w_j(k) v(k)] = 0 \text{ for all } j \quad (2.8b)$$

That is, it is assumed that the noise perturbations associated with one coefficient are independent of the noise perturbations associated with any other coefficient, and that the measurement noise and the coefficient perturbations are independent.

At this point it is necessary to use judgment and experience and utilize all the information known about the physical system which equation (2.7) represents to assign variances for the random processes  $\{w_i(k)\}$ ,  $\{w_j(k)\}$  and  $\{v(k)\}$ . Let

$$Q = E \left\{ \begin{bmatrix} w_i(k) \\ w_j(k) \end{bmatrix} [w_i(k) \ w_j(k)] \right\} \quad (2.9a)$$

$$Q = \begin{bmatrix} Q_1 & 0 \\ 0 & Q_2 \end{bmatrix} \quad (2.9b)$$

where,  $Q_1 = \text{diag } (\sigma_{w_i}^2)$ ,  $Q_2 = \text{diag } (\sigma_{w_j}^2)$  and,

$$R = E[v(k) v(k)] \quad (2.9c)$$

and  $E[x]$  is the expected or average value of  $x$ .

Instead of using  $x$ 's denoting the commonly used notation for state variables, we retain the flavor of the problem by using the  $a_i$ 's and  $b_i$ 's as the "states" of the Adaptive Kalman Identifier. It is hoped that a more meaningful understanding of the Adaptive Kalman Identifier may thus be gained. Therefore, define the state vector,

$$\begin{bmatrix} a_0(k) \\ a_1(k) \\ \vdots \\ a_m(k) \\ \hline b_1(k) \\ b_2(k) \\ \vdots \\ b_n(k) \end{bmatrix} = \begin{bmatrix} \underline{a} \\ \hline \underline{b} \end{bmatrix} \quad (2.10)$$

Combining equations (2.5) and (2.10), we have the discrete, first order Gauss-Markov process [Ref. 3],

$$\begin{bmatrix} \underline{a}(k+1) \\ \hline \underline{b}(k+1) \end{bmatrix} = \begin{bmatrix} \underline{a}(k) \\ \hline \underline{b}(k) \end{bmatrix} + \underline{w}(k) \quad (2.11)$$

where,

$$\underline{w}(k) = \begin{bmatrix} w_i(k) \\ \hline \vdots \\ w_j(k) \end{bmatrix} . \quad (2.12)$$

Note that  $\underline{w}(k)$  is an  $n+m+1$  vector whose elements are a concatenation of the noise sequences associated with the  $a_i$  and  $b_i$ . To complete the Adaptive Kalman Identifier formulation we define the measurement vector,

$$H(k) = [u(k) \ u(k-1) \ \dots \ u(k-m) \ -y(k-1) \ \dots \ -y(k-n)] \quad (2.13)$$

and its associated measurement equation,

$$y(k) = H(k) \begin{bmatrix} \underline{a}(k) \\ \hline \underline{b}(k) \end{bmatrix} + v(k). \quad (2.14)$$

Then the solution to the Adaptive Kalman Identifier problem [Ref. 21] is,

$$\begin{bmatrix} \hat{\underline{a}}(k+1|k) \\ \hline \hat{\underline{b}}(k+1|k) \end{bmatrix} = [I - K(k)H(k)] \begin{bmatrix} \hat{\underline{a}}(k|k-1) \\ \hline \hat{\underline{b}}(k|k-1) \end{bmatrix} + K(k)y(k) \quad (2.15a)$$

$$K(k) = P(k|k-1) H^T(k) [H(k) P(k|k-1) H^T(k) + R]^{-1} \quad (2.15b)$$

$$P(k+1|k) = P(k|k-1) - K(k)H(k)P(k|k-1) + Q. \quad (2.15c)$$

Equations (2.15) are initialized by assuming an initial value for the coefficients (2.10) and assigning to our assumption a measure of our confidence in the initial guess. That is, we pick the values,

$$\begin{bmatrix} \hat{\underline{a}}(0|-1) \\ \hline \hat{\underline{b}}(0|-1) \end{bmatrix} \text{ and } P(0|-1) \quad (2.16)$$

where  $P(0|-1)$  is defined as the error covariance of the coefficients,

$$P(k|k-1) = E \left\{ \begin{bmatrix} \underline{a}(k) \\ \vdots \\ \underline{b}(k) \end{bmatrix} - \begin{bmatrix} \hat{\underline{a}}(k|k) \\ \vdots \\ \hat{\underline{b}}(k|k) \end{bmatrix} \begin{bmatrix} \underline{a}(k) \\ \vdots \\ \underline{b}(k) \end{bmatrix}^T - \begin{bmatrix} \hat{\underline{a}}(k|k) \\ \vdots \\ \hat{\underline{b}}(k|k) \end{bmatrix} \right\}^T \quad (2.17)$$

for  $k = 0$ . Equation (2.15b) is generally referred to as the Kalman gain. Two special cases are of interest: case (1);  $a_i = 0$  for all  $i$ , and case (2);  $b_i = 0$  for all  $i$ .

### 1. Autoregressive Form ( $a_i = 0$ )

For case 1, equation (2.7) takes the form,

$$y(k) = - \sum_{i=1}^n b_i y(k-i) + v(k). \quad (2.18)$$

This is a recursive equation stating that the present output is a linear combination of past outputs corrupted with additive gaussian noise,  $v(k)$ . Equation (2.18) is more formally recognized as an autoregressive process of order  $n$  [Ref. 25, 26]. Writing the Kalman solution for the coefficients of the AR process,

$$\hat{\underline{b}}(k+1|k) = [I - K(k)K(k)] \hat{\underline{b}}(k|k-1) + K(k)y(k) \quad (2.19a)$$

$$K(k) = P(k|k-1) H^T(k) [H(k)P(k|k-1) H^T(k) + R]^{-1} \quad (2.19b)$$

$$P(k+1|k) = P(k|k-1) - K(k)H(k)P(k|k-1) + Q. \quad (2.19c)$$

Alternate forms of the Kalman equations given by Maybeck [Ref. 27] are,

$$\begin{aligned} \hat{\underline{b}}(k+1|k) &= P(k|k-1) P(k|k-1)^{-1} \hat{\underline{b}}(k|k-1) \\ &\quad + P(k+1|k) H^T(k) R^{-1} y(k) \end{aligned} \quad (2.20a)$$

$$P(k+1|k) = (P(k|k-1))^{-1} + H^T(k) R^{-1} H(k)^{-1}. \quad (2.20b)$$

We can model the fact that we have no a-priori knowledge about the initial values of the coefficients by letting,

$$[P(0|-1)]^{-1} \approx 0. \quad (2.21)$$

Further, if we remain ignorant and totally doubt our previous estimate, then  $P(k|k-1)$  is modeled as,

$$[P(k|k-1)]^{-1} \approx 0. \quad (2.22)$$

Equations (2.20) reduce to,

$$\hat{\underline{b}}(k+1|k) = [H(k) R^{-1} H^T(k)]^{-1} H(k) R^{-1} Y(k), \quad (2.23)$$

the weighted least squares estimate, previously encountered by Swerling [Ref. 28,29,30], of the coefficients,  $\underline{b}$ . Carrying the analysis further and letting  $R^{-1} = I$ , we have the Penrose pseudo-inverse solution [Ref. 31,32],

$$\hat{\underline{b}}(k+1|k) = [H(k) H^T(k)]^{-1} H(k) Y(k) \quad (2.24)$$

Swerling [Ref. 28] has shown that if the weighting matrix in (2.23) is not the inverse of the covariance matrix of the measurement errors, then the accuracy of the estimated coefficients (2.24) will be degraded.

The aforementioned notwithstanding, we press further into the analysis of equation (2.24). The product  $H(k)H^T(k)$  can be written as,

$$H(k)H^T(k) = \begin{bmatrix} y(k-1) \\ y(k-2) \\ \vdots \\ y(k-n) \end{bmatrix} [y(k-1) \ y(k-2) \ \dots \ y(k-n)] \quad (2.25)$$

$$H(k)H^T(k) = \begin{bmatrix} y^2(k-1) & y(k-1)y(k-2) & \dots & y(k-1)y(k-n) \\ y(k-2)y(k-1) & y^2(k-2) & & \vdots \\ \vdots & & & \vdots \\ y(k-n)y(k-1) & \dots & \dots & y(k-n)y(k-n) \end{bmatrix}. \quad (2.26)$$

As we let  $k \rightarrow \infty$  the expected value of (2.26) becomes the autocorrelation matrix,

$$E\{\lim_{k \rightarrow \infty} H(k) H^T(k)\} = R_{yy}(k) \quad (2.27)$$

where,

$$R_{YY}^{(k)} = \begin{bmatrix} R_{YY}^{(0)} & R_{YY}^{(1)} & \dots & R_{YY}^{(n)} \\ R_{YY}^{(-1)} & R_{YY}^{(0)} & \dots & \vdots \\ \vdots & & & \vdots \\ R_{YY}^{(-n)} & \dots & \dots & R_{YY}^{(0)} \end{bmatrix} . \quad (2.28)$$

Similarly, the product  $H(k)y(k)$  as  $k \rightarrow \infty$  becomes

$$r_{yy}^{(k)} = \begin{bmatrix} R_{YY}^{(-1)} \\ R_{YY}^{(-2)} \\ \vdots \\ R_{YY}^{(-n)} \end{bmatrix} . \quad (2.29)$$

The steady state solution for the estimate of the coefficients  $\hat{b}_{ss}(k+1|k)$  is,

$$\hat{b}_{ss}(k+1|k) = R_{YY}^{-1}(k)r_{yy}^{(k)} . \quad (2.30)$$

Equation (2.30) is one of the starting points from which Perry [Ref. 6] develops his Lattice modeling algorithms. Equation (2.30) can also be recognized as the solution to the Yule-Walker or Normal equations [Ref. 26].

## 2. Moving Average Form ( $b_i = 0$ )

Referring once again to equation (2.7) and setting the coefficients  $b_i = 0$  for all  $i$  we have case (2),

$$y(k) = \sum_{i=0}^m u(k-i) + v(k) \quad (2.31)$$

Since  $v(k)$  is by definition noise associated with the measurement, we can combine  $y(k)$  and  $v(k)$  such that,

$$z(k) = y(k) - v(k) = \sum_{i=0}^m a_i u(k-i). \quad (2.32)$$

Equation (2.32) simply states that the present measurement is a linear combination of past and present inputs or by definition, a Moving Average (MA) process [Ref. 25,33]. The Adaptive Kalman Identifier estimate for the coefficients of the MA process is as before,

$$\hat{a}(k+1|k) = [I - K(k)H(k)] \hat{a}(k+1|k) + K(k)z(k) \quad (2.33a)$$

$$K(k) = P(k|k-1) H^T(k) [H(k)P(k|k-1) H^T(k) + R]^{-1} \quad (2.33b)$$

$$P(k+1|k) = P(k|k-1) - K(k)H(k)P(k|k-1) + Q. \quad (2.33c)$$

Using the alternate forms of the above equations we arrive at,

$$\begin{aligned} \hat{a}(k+1|k) &= [P(k+1|k) [P(k|k-1)]^{-1}] \hat{a}(k|k-1) \\ &\quad + [P(k+1|k) H^T(k) R^{-1}] z(k) \end{aligned} \quad (2.34a)$$

$$P(k+1|k) = [(P(k|k-1))^{-1} + H^T(k) R^{-1} H(k)]^{-1}. \quad (2.34b)$$

Arguing as we did for the autoregressive case, no a-priori knowledge about the initial values of the moving average coefficients, implies that,

$$[P(0|-1)]^{-1} \approx 0. \quad (2.35)$$

And even if after we processed one measurement we still admitted no knowledge as to the accuracy of the previous estimate, we imply that,

$$[P(k|k-1)]^{-1} \approx 0. \quad (2.36)$$

Substituting these implications into equations (2.34) our estimate becomes,

$$\hat{a}(k+1|k) = [H(k) R^{-1} H^T(k)]^{-1} H(k) R^{-1} z(k) \quad (2.37)$$

the by now familiar weighted least squares estimate [Ref. 28, 29, 30]. If once again we allow  $R^{-1}$  to be unity, let  $k$  tend toward infinity and take the expectation of equation (2.37), one arrives at the discrete form of the Wiener-Hopf equation [Ref. 11], namely

$$\hat{a}(k+1|k) = R_{uu}^{-1}(k) r_{uz}(k) \quad (2.38)$$

where it can easily be shown that,

$$R_{uu}(k) = \begin{bmatrix} R_{uu}(0) & R_{uu}(-1) & \dots & R_{uu}(-m) \\ R_{uu}(-1) & R_{uu}(0) & \dots & R_{uu}(1-m) \\ \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ R_{uu}(-m) & \dots & & R_{uu}(0) \end{bmatrix} \quad (2.39)$$

and,

$$R_{uy}(k) = \begin{bmatrix} R_{uy}(0) \\ R_{uy}(-1) \\ \vdots \\ \vdots \\ R_{uy}(-m) \end{bmatrix} . \quad (2.40)$$

Equation (2.38) provided another point of departure from which Perry [Ref. 6] develops the MA Lattice modeling algorithms. Appendix A develops the Wiener-Hopf equation otherwise known as the all-zero model from yet another approach.

The all-zero model is fundamental to Widrow least mean square (LMS) adaptive filters [Ref. 11] and linear prediction theory [Ref. 16].

### 3. Autoregressive-Moving Average Form

Returning to the alternate form (equations 2.20) of the Kalman Filter equations (2.19) the development of the Autoregressive Moving Average (ARMA) Adaptive Kalman Identifier follows. The estimate for the  $a_i$  and  $b_i$  coefficients is,

$$\begin{bmatrix} \hat{a}(k+1|k) \\ \hat{b}(k+1|k) \end{bmatrix} = [P(k+1|k) [P(k|k-1)]^{-1}] \begin{bmatrix} \hat{a}(k|k-1) \\ \hat{b}(k|k-1) \end{bmatrix} + [P(k+1|k) H^T(k) R^{-1}] y(k) \quad (2.41a)$$

$$P(k+1|k) = [(P(k|k-1))^{-1} + H^T(k) R^{-1} H(k)]^{-1} . \quad (2.41b)$$

Progressing in the same manner as in the previous two cases, we assume,

$$[P(0|-1)]^{-1} \approx 0 \quad (2.42a)$$

$$[P(k|k-1)]^{-1} \approx 0. \quad (2.42b)$$

The estimate then becomes,

$$\begin{bmatrix} \hat{a}(k+1|k) \\ \hat{b}(k+1|k) \end{bmatrix} = [H^T(k) R^{-1} H(k)]^{-1} H(k) R^{-1} y(k). \quad (2.43)$$

Taking equation (2.43) a step further by letting  $R^{-1}$  be unity, letting  $k$  approach infinity and taking the expectation we have,

$$\begin{bmatrix} \hat{a}(k+1|k) \\ \hat{b}(k+1|k) \end{bmatrix} = \begin{bmatrix} R_{nu}(k) & -R_{uy}(k-1) \\ -R_{uy}^T(k-1) & R_{yy}(k) \end{bmatrix}^{-1} \begin{bmatrix} R_{uy}(k) \\ R_{yy}(k) \end{bmatrix} \quad (2.44)$$

Note that the time varying measurement vector,  $H(k)$ , is of the form

$$H(k) = [u(k) u(k-1) \dots u(n-m) | -y(k-1) -y(k-2) \dots -y(k-n)] \quad (2.45)$$

Shown in detail, equation (2.44) has the characteristic form (equation 2.46, next page). It is the Toeplitz and symmetric nature of equation (2.46) that is exploited by the Levinson [Ref. 34] and Lattice [Ref. 35] algorithms.

$$\begin{bmatrix}
R_{uu}(0) & R_{uu}(-1) & \cdots & R_{uu}(-m) \\
R_{uu}(-1) & R_{uu}(0) & \cdots & R_{uu}(1-m) \\
\vdots & \vdots & \ddots & \vdots \\
\vdots & \vdots & \ddots & \vdots \\
R_{uu}(1-m) & \cdots & R_{uu}(0) & -R_{uy}(m-1) \cdots -R_{uy}(m-n) \\
\hline
\hat{\underline{a}}(k+1|k) \\
\hat{\underline{b}}(k+1|k)
\end{bmatrix} = \begin{bmatrix}
-R_{uy}(-1) & -R_{uy}(-2) & \cdots & -R_{uy}(-n) \\
-R_{uy}(0) & -R_{uy}(1) & \cdots & -R_{uy}(1-n) \\
\vdots & \vdots & \ddots & \vdots \\
\vdots & \vdots & \ddots & \vdots \\
-R_{yu}(-1) & -R_{yu}(0) & \cdots & -R_{yu}(m-1) \\
-R_{yu}(-2) & -R_{yu}(-1) & \cdots & \vdots \\
\vdots & \vdots & \ddots & \vdots \\
\vdots & \vdots & \ddots & \vdots \\
-R_{yu}(-n) & \cdots & \cdots & -R_{yu}(m-n) \\
\hline
R_{yy}(0) & R_{yy}(-1) & \cdots & R_{yy}(-n) \\
R_{yy}(-1) & R_{yy}(0) & \cdots & \vdots \\
\vdots & \vdots & \ddots & \vdots \\
\vdots & \vdots & \ddots & \vdots \\
R_{yy}(-n) & \cdots & \cdots & R_{yy}(0)
\end{bmatrix}^{-1} \begin{bmatrix}
R_{uy}(0) \\
R_{uy}(1) \\
\vdots \\
\vdots \\
R_{uy}(m)
\end{bmatrix}$$

(2.46)

The Adaptive Kalman ARMA modeling technique can be best visualized by a block diagram. Referring to Figure 2.1, the unknown system is excited by a white, zero mean, gaussian noise sequence of sample values from a random process. The input and its associated output are then passed through M and N delays respectively in serial form. The parallel inputs ( $M+1$ ) and the outputs (N) are concatenated to form the measurement vector  $H(k)$  which is represented by equation (2.13). The inputs N and M are selected a-priori as the model orders for the Autoregressive and Moving Average processes one desires to identify. It can be easily seen that the remainder of the figure simply implements equation (2.15a). The outputs of the Adaptive Kalman Identifier are an estimate of the coefficients,

$$\begin{bmatrix} \hat{b}(k|k) \\ \hline \hat{a}(k|k) \end{bmatrix}$$

and a one step prediction,  $\hat{y}(k|k)$ .

#### C. OBSERVATIONS

In this section it has been shown that a direct connection can be established between the Adaptive Kalman Identifier and the Yule-Walker equations associated with an AR process. Secondly, the steady state Adaptive Kalman Identifier closely resembles the discrete Weiner-Hopf equation associated with the MA process when the measurement matrix,  $H(k)$ , is of the form,

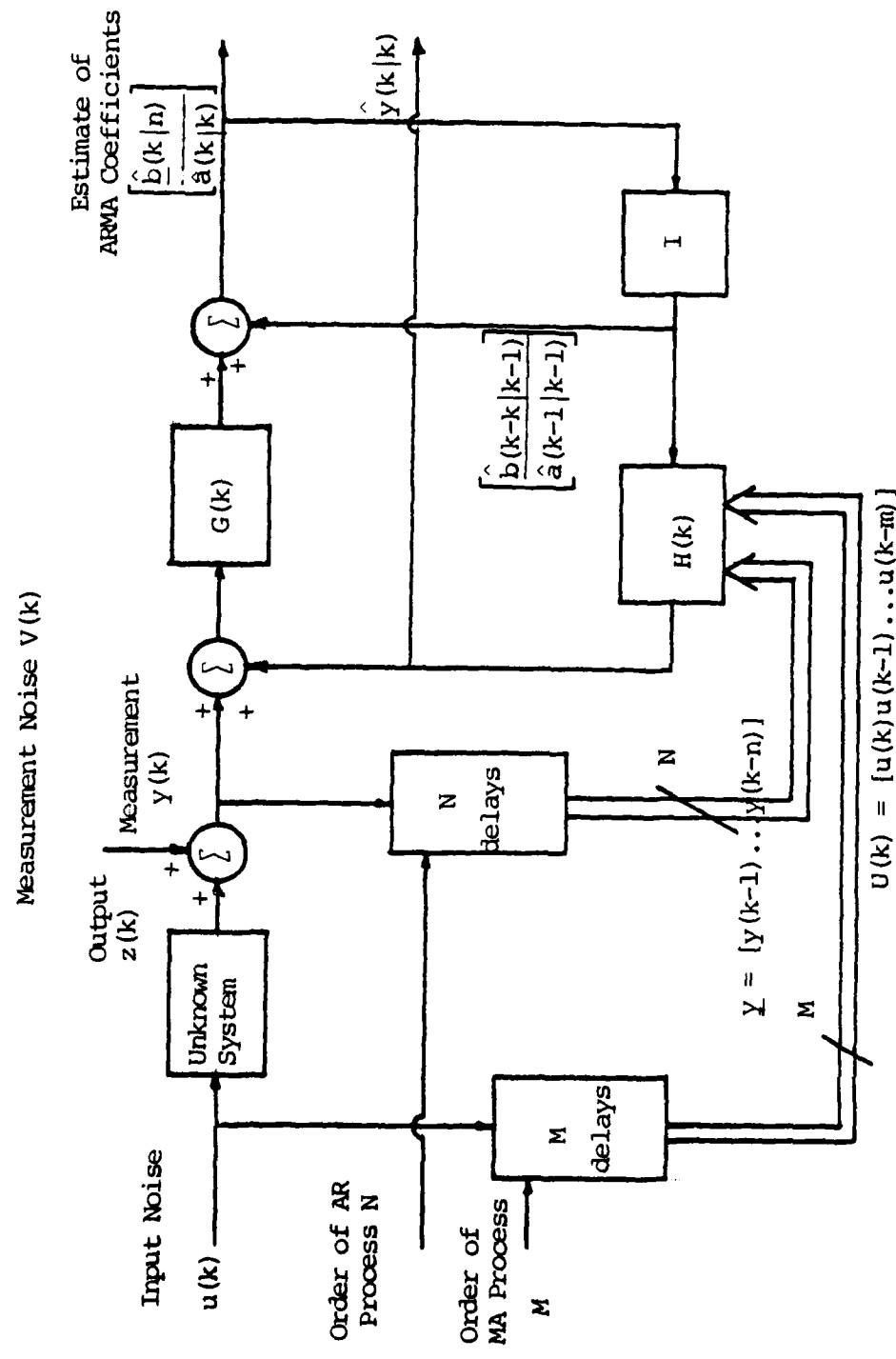


Figure 2.1. Adaptive Kalman Identifier

$$H(k) = [u(k) \ u(k-1) \ u(k-2) \ \dots \ u(k-m)]$$

And, thirdly under the same assumptions as in the previous two cases, the Adaptive Kalman ARMA Identifier is similar to the form Perry [Ref. 6] exploits using Lattice modeling.

### III. COMPARISON BETWEEN THE ADAPTIVE MA KALMAN IDENTIFIER AND THE WIDROW LMS ADAPTIVE FILTER

#### A. PRELIMINARIES

It is instructive to investigate the similarities between the Adaptive Kalman Identifier and the Widrow LMS Adaptive Filter when the concepts are applied to system identification. Basically, the LMS algorithm implementation of system identification considers a block diagram as is shown in Figure 3.1. The output,  $y(k)$ , of the Adaptive filter is simply a weighted linear combination of the past and present known inputs. The same input is fed into both the unknown system to be identified and the adaptive filter. The output of the unknown system is designated the desired response,  $d(k)$ , from which an error signal,  $\epsilon(k)$ , is derived. The error signal,  $\epsilon(k)$ , provides the criterion through which the weights,  $w_i$ , are adjusted such that the error,  $\epsilon(k+1)$ , is driven toward its minimum. A more detailed analysis of the operation of the LMS algorithm can be found in Ref. 11.

The weights  $w_i$  are adjusted from time step to time step in the following manner,

$$\underline{w}(k+1) = \underline{w}(k) - 2k_s \epsilon(k) \underline{x}(k) \quad (3.1)$$

where we define the following quantities,

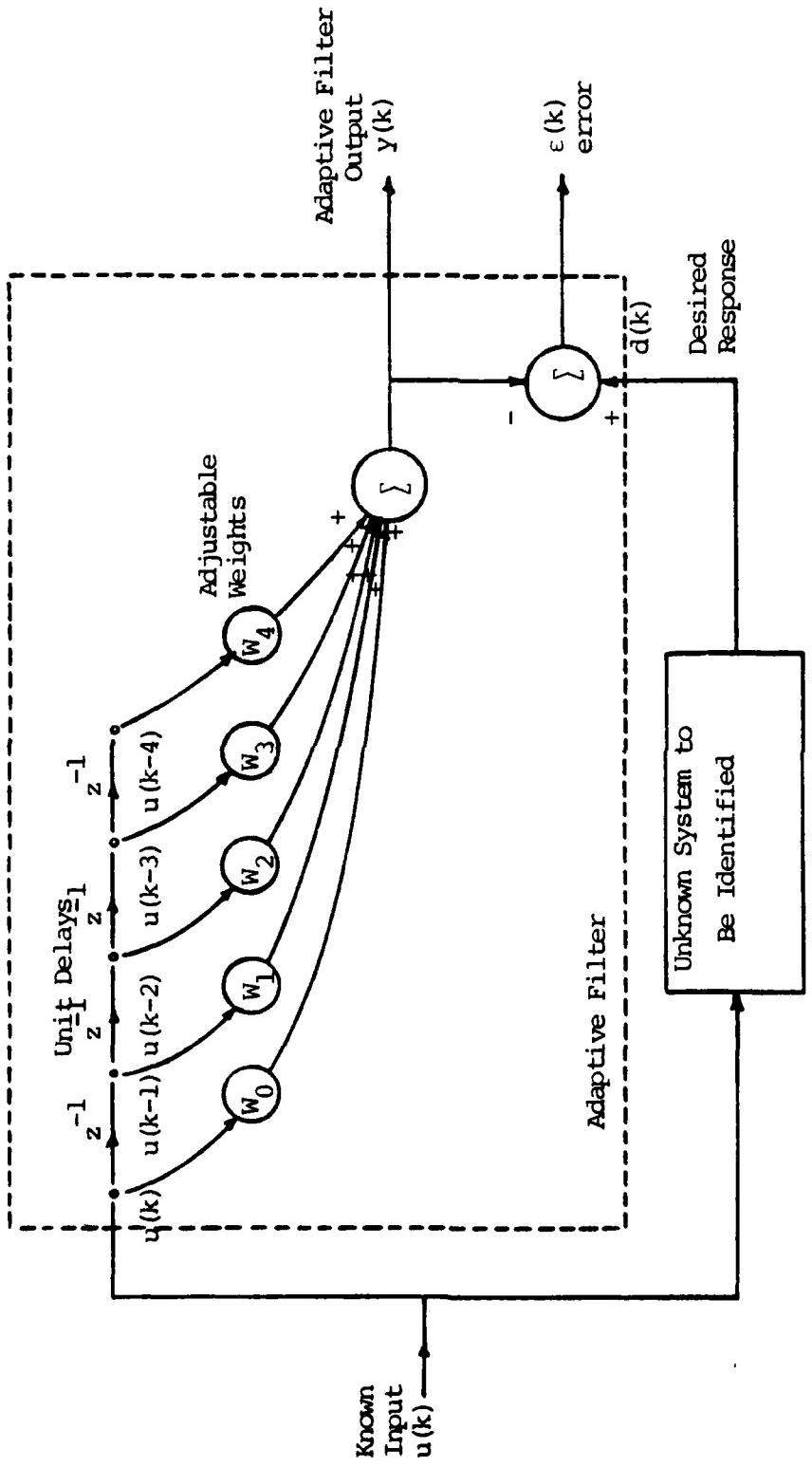


Figure 3.1. LMS Adaptive Filter Implementation of System Identification

$$\underline{w}(k) = \begin{bmatrix} w_0(k) \\ w_1(k) \\ \vdots \\ \vdots \\ w_m(k) \end{bmatrix} \quad (3.2)$$

the weight vector at time step  $k$ ,

$$\underline{x}(k) = \begin{bmatrix} u(k) \\ u(k-1) \\ \vdots \\ \vdots \\ u(k-m) \end{bmatrix} \quad (3.3)$$

the input signal vector at time step  $k$ ,

$$\varepsilon(k) = d(k) - y(k) \quad (3.4)$$

$$y(k) = \underline{w}^T(k) \underline{x}(k) \quad (3.5)$$

the error,  $\varepsilon(k)$ , and the output,  $y(k)$ , at time step  $k$ , and,  $k_s$ , a scalar constant controlling the rate of convergence and the stability of the adaptive filter.

#### B. THE COMPARISON

Substituting (3.4) and (3.5) into (3.1) gives,

$$\underline{w}(k+1) = \underline{w}(k) - 2k_s \underline{x}(k) [d(k) - \underline{w}^T(k) \underline{x}(k)]. \quad (3.6)$$

Recalling the MA form of the Adaptive Kalman Identifier, equation (2.33a), rewritten here for convenience as

$$\hat{\underline{a}}(k+1|k) = \hat{\underline{a}}(k|k-1) + K(k) [z(k) - H(k) \hat{\underline{a}}(k|k-1)], \quad (3.7)$$

one can make the following associations:

$$\hat{\underline{a}}(k+1|k) \Leftrightarrow \underline{w}(k+1) \quad (3.8a)$$

$$\hat{\underline{a}}(k|k-1) \Leftrightarrow \underline{w}(k) \quad (3.8b)$$

$$z(k) \Leftrightarrow d(k) \quad (3.8c)$$

$$H^T(k) \Leftrightarrow \underline{X}(k) \quad (3.8d)$$

$$K(k) \Leftrightarrow -2k_s \underline{X}(j) \quad (3.8e)$$

Recall that for the MA form of the AKI, the measurement vector  $H(k)$  represents a vector of past and present inputs. Namely,

$$H(k) = [u(k) \ u(k-1) \ \dots \ u(k-m)]. \quad (3.9)$$

Therefore the associations (3.8a)-(3.8d) are straightforward. However, it is not so clear as to what is meant by the association (3.8e). Digressing a moment to present an equivalent expression [Ref. 27] for the Kalman gain,  $K(k)$ ,

$$K(k) = P(k|k) H^T(k) R^{-1} \quad (3.10)$$

and substituting (3.10) into the association (3.8e) and entertain the conjecture that the quantities are equivalent

under certain conditions, we have:

$$P(k|k) H^T(k) R^{-1} = -2k_s \underline{X}(k) . \quad (3.11)$$

The conditions alluded to are (1) that the Adaptive Kalman Identifier is in steady state and (2) that the statistics of the input forcing function are stationary. Denoting the steady state error covariance,  $P(k|k)$ , as  $P_\infty$ , equation (3.11) can be solved for  $k_s$ ,

$$k_s I = -\frac{1}{2} R^{-1} P_\infty . \quad (3.12)$$

Invoking the entire Kalman gain history in its more popular form of equation (2.33b) and equating it to the Widrow gain (3.8e), we instead arrive at,

$$-2k_s \underline{X}(k) = P(k|k-1) H^T(k) [H(k) P(k|k-1) H^T(k) + R]^{-1} \quad (3.13a)$$

$$H^T(k) = \underline{X}(k) . \quad (3.13b)$$

Solving for the convergence factor,  $k_s$ , we obtain,

$$\begin{aligned} -2k_s \underline{X}(k) \underline{X}^T(k) &= P(k|k-1) \underline{X}(k) [\underline{X}^T(k) P(k|k-1) \underline{X}(k) + R]^{-1} \underline{X}^T(k) \\ -2k_s I &= P(k|k-1) \underline{X}(k) [\underline{X}^T(k) P(k|k-1) \underline{X}(k) + R]^{-1} \underline{X}^T(k) [\underline{X}(k) \underline{X}^T(k)]^{-1}. \end{aligned}$$

But  $[\underline{X}^T(k) P(k|k-1) \underline{X}(k) + R]^{-1}$  is a scalar. Therefore,

$$-2k_s I = \frac{P(k|k-1)}{\underline{X}^T(k) P(k|k-1) \underline{X}(k) + R} \quad (3.14)$$

Griffiths [Ref. 22] suggests that the convergence factor  $k_s$ , which he denotes as  $\mu(n)$ , should be chosen such that,

$$\mu(n) = \frac{\alpha}{L \hat{\sigma}_x^2(n)} \quad (3.15)$$

where  $0 < \alpha < 1$  is a normalized adaptive stepsize parameter and the term  $\hat{\sigma}_x^2(n)$  is an estimate of the input power level which may be computed using a geometrically-weighted average for an L weight adaptive filter,

$$\hat{\sigma}_x^2(n) = (1 - \frac{\alpha}{L}) \hat{\sigma}_x^2(n-1) + \frac{\alpha}{L} x^2(n) . \quad (3.16)$$

Comparing equation (3.14) and (3.15), one observes that if  $P(k|k-1)$  is equal to the identity matrix then we obtain a term proportional to the input power. Further if measurement noise is considered negligible,  $R \rightarrow 0.0$ , then 3.14 and 3.15 are essentially equivalent. The salient point to make, however, is that the choice of the convergence constant,  $k_s$ , in the LMS algorithm provides no clue as to how to deal with measurement noise whereas the Adaptive MA Kalman Identifier explicitly incorporates measurement error into its algorithm.

### C. OBSERVATIONS

It has been known that the LMS algorithm was suboptimal since the actual gradient of equation (3.1) is replaced by the approximation,  $2\varepsilon(k) x(k)$ , [Ref. 24]. However, the degree of suboptimality is difficult to quantify. In previous linear prediction research no mention was made regarding the

role of measurement noise. Equation (3.14) gives us some insight into wiser selections of the rate of convergence constant,  $k_s$ , since it takes into account the effects of measurement noise. Further, it appears that the LMS adaptive filter is a special case of the Adaptive MA Kalman Identifier.

#### IV. COMPARISON BETWEEN THE ADAPTIVE ARMA KALMAN IDENTIFIER AND THE ADAPTIVE RECURSIVE LMS FILTER

This chapter endeavors to investigate the similarities and differences between the Adaptive ARMA Kalman Identifier and the Adaptive Recursive LMS Filter [Ref. 18]. LMS filters [Ref. 11] have enjoyed much popularity in the recent past due to their ease of implementation, simple unimodal algorithm, robustness and ability to "adapt" to the unknown statistics of the signal environment. Being transversal in nature, they have a finite impulse response being able to produce only zeros in the input/output transfer function. One may, however, decide to model the transfer function,

$$H_1(z) = \frac{1}{1 - .9z^{-1}} \quad (4.1)$$

using a transversal filter, only to realize that a large number of delays are required in order to arrive at an adequate approximation. The germane point which is being made is that one weighted feedback tap can realize an infinite string of feed-forward coefficients. Moreover, it is very desirous to adjust the feedback weights adaptively in some optimal fashion to the statistics of the signal environment.

##### A. PRELIMINARIES

The approach taken by Feintuch [Ref. 18] is patterned after the analysis first presented by Widrow [Ref. 11]. A summary of Feintuch's derivation will be presented here with emphasis on its application to ARMA modeling.

Recall the general ARMA equation (2.1b),

$$y(k) = \sum_{i=0}^m a_i u(k-i) + \sum_{i=1}^n b_i y(k-i) \quad (4.2a)$$

which can be rewritten in vector notation,

$$\underline{y}(k) = \underline{a}^T \underline{u} + \underline{b}^T \underline{y} \quad (4.2b)$$

where,

$$\underline{a}^T = [a_0 \ a_1 \ \dots \ a_m] \quad (4.3a)$$

$$\underline{b}^T = [b_1 \ b_2 \ \dots \ b_n] \quad (4.3b)$$

$$\underline{y} = [y(k-1) \ y(k-2) \ \dots \ y(k-n)]^T \quad (4.3c)$$

$$\underline{u} = [u(k) \ u(k-1) \ \dots \ u(k-m)]^T \quad (4.3d)$$

Given that (4.2) is the assumed mathematical description of the unknown system where  $\underline{u}$  and  $\underline{y}$ , the input and output data sequences, are known, then by solving (4.2) for  $\underline{a}$  and  $\underline{b}$ , identification of the unknown system can be made. The LMS algorithms, in general, employ a "desired" signal  $d(k)$  with which to "train" the adaptive filter. If the desired signal was assumed to be the response of some unknown system to a known input signal, then the algorithm presented by Feintuch [Ref. 18] can be used as a means by which to identify the system parameters.

The first step in the Recursive LMS derivation is to form the error between the desired signal,  $d(k)$ , and the output

of the filter,  $y(k)$ ,

$$e(k) = d(k) - y(k) = d(k) - \underline{a}^T \underline{u} - \underline{b}^T \underline{y} \quad (4.4)$$

Forming the square of (4.4) and taking the expectation, we have the mean square error representation of the filter,

$$\begin{aligned} E\{e^2(k)\} &= E\{d^2(k)\} + \underline{a}^T R_{uu} \underline{a} + \underline{b}^T R_{yy} \underline{b} \dots \\ &\quad - 2\underline{a}^T R_{du} - 2\underline{b}^T R_{dy} + 2\underline{a}^T R_{uy} \underline{b} \end{aligned} \quad (4.5)$$

where,

$$R_{uu}(k) = E\{\underline{u}(k) \underline{u}^T(k)\} \quad (4.6a)$$

$$R_{yy}(k-1) = E\{\underline{y}(k-1) \underline{y}^T(k-1)\} \quad (4.6b)$$

$$R_{du}(k) = E\{d(k) \underline{u}(k)\} \quad (4.6c)$$

$$R_{dy}(k) = E\{d(k) \underline{y}(k)\} \quad (4.6d)$$

$$R_{uy}(k-1) = E\{\underline{u}(k-1) \underline{y}(k-1)\}. \quad (4.6e)$$

It is assumed that the statistics (4.6) are constants allowing the gradient of (4.5) to be taken with respect to the  $a_i$  and  $b_i$ . This assumption does not stretch the theory since in practice one uses an input test signal with stationary characteristics and if the unknown system's output process is not stationary, then no identification of the system parameters can be made. The respective gradients are,

$$\frac{\partial}{\partial \underline{a}} [E\{e^2(k)\}] = 2R_{uu}(k)\underline{a} - 2R_{du}(k) + 2R_{uy}(k)\underline{b} \quad (4.7a)$$

$$\frac{\partial}{\partial \underline{b}} [E\{e^2(k)\}] = 2R_{yy}(k)\underline{b} - 2R_{dy}(k) + 2R^T_{uy}(k)\underline{a}. \quad (4.7b)$$

Since the second order statistics are assumed to be known, equations (4.7) can be solved for  $\underline{a}$  and  $\underline{b}$  by setting the gradients equal to zero. In matrix partitioned form we have,

$$\begin{bmatrix} R_{uu}(k) & | & R_{uy}(k) \\ \hline R^T_{uy}(k) & | & R_{yy}(k) \end{bmatrix} \begin{bmatrix} \underline{a} \\ \hline \underline{b} \end{bmatrix} = \begin{bmatrix} R_{du}(k) \\ \hline R_{dy}(k) \end{bmatrix}. \quad (4.8)$$

At this point we digress momentarily to mention that Johnson and Larimore [Ref. 19] and Widrow and McCool [Ref. 20] agree with Feintuch's derivation. The following steps, however, are controversial [Ref. 19,20].

Feintuch continues and states that in general, the statistics involved in (4.8) are not available a-priori. One method of estimating the statistics is to make the filter adaptive in an iterative fashion using the method of steepest descent. The method of steepest descent employs an algorithm of the form,

$$\underline{a}(k+1) = \underline{a}(k) + k_a \frac{\partial}{\partial \underline{a}} [E\{e^2(k)\}] \quad (4.9a)$$

$$\underline{b}(k+1) = \underline{b}(k) + k_b \frac{\partial}{\partial \underline{b}} [E\{e^2(k)\}]. \quad (4.9b)$$

The gradient involved in (4.9) can be approximated using the techniques outlined in [Ref. 11]. The iterative LMS algorithm

for estimating the coefficients  $\underline{a}$  and  $\underline{b}$  is,

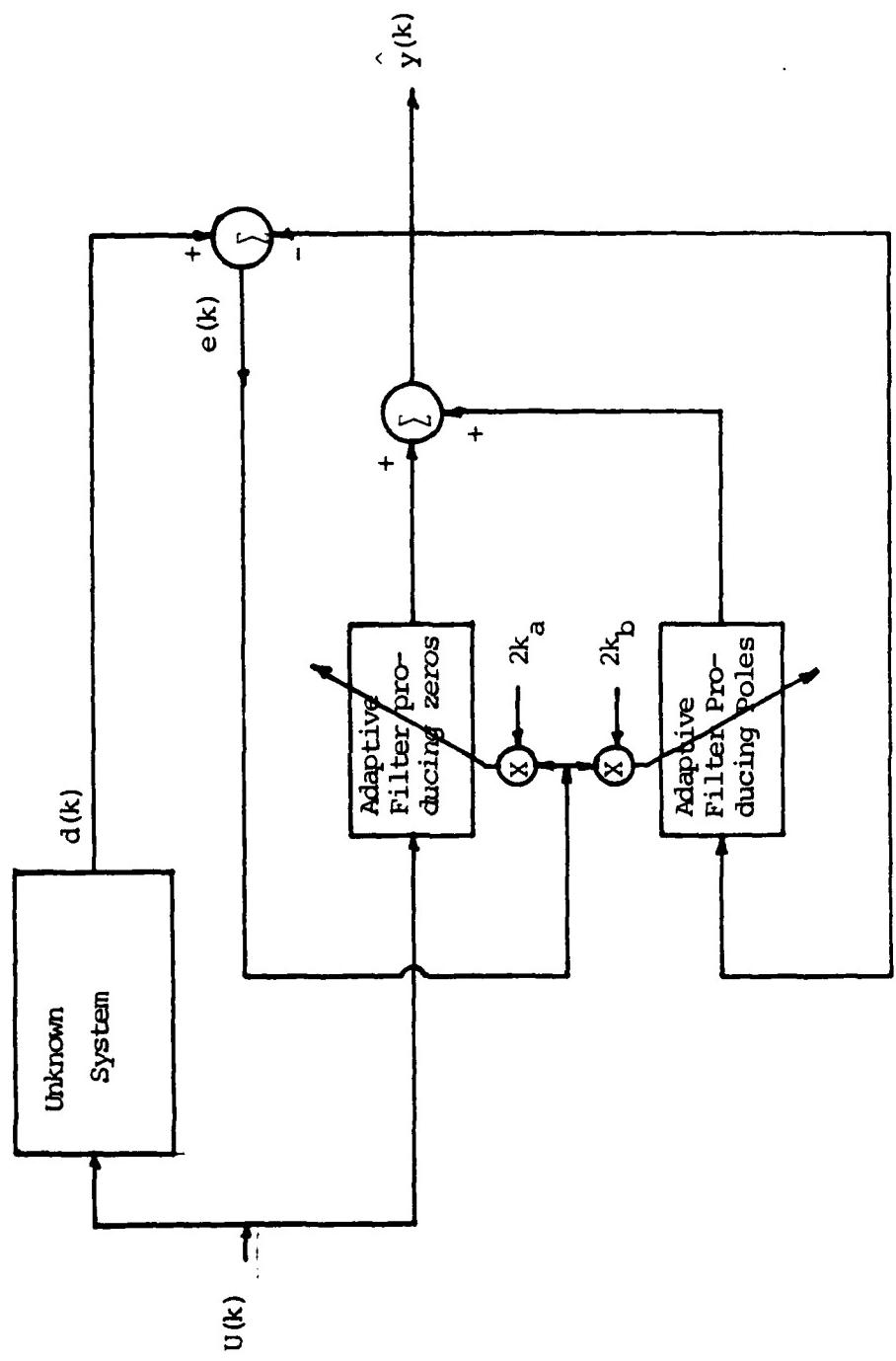
$$\underline{a}(k+1) = \underline{a}(k) - 2k_a e(k) \underline{u}(k) \quad (4.10a)$$

$$\underline{b}(k+1) = \underline{b}(k) - 2k_b e(k) \underline{y}(k). \quad (4.10b)$$

Figure 4.1 is a block diagram implementation of the Adaptive Recursive LMS Filter algorithm as applied to system identification. The input signal,  $u(k)$ , to both the unknown system and the adaptive filter is a zero mean, white gaussian noise sequence of samples from a random process. The output of the unknown system is  $d(k)$  which is used as the training signal for the filter. The output of the adaptive filter,  $y(k)$ , is compared with the desired signal,  $d(k)$ , from which an error signal is derived. The error signal,  $e(k)$ , is then used in the LMS algorithm, equation (4.10), to iteratively adjust the  $\underline{a}$  and  $\underline{b}$  coefficients. Theoretically, when the coefficients of the zeros-producing adaptive filter and the poles-producing adaptive filter correspond exactly with those of the unknown system, the error sequence should be zero and the unknown system is modeled by the Adaptive Recursive LMS Filter. The convergence constants,  $k_a$  and  $k_b$ , are chosen a-priori as are the initial values,  $\underline{a}(0)$  and  $\underline{b}(0)$ , for the coefficients.

#### B. THE COMPARISON

One can begin the comparison between the Adaptive Kalman Identifier and the Adaptive Recursive LMS filter by noting



Filter 4.1. Adaptive Recursive LMS Filter Applied to System Identification

the similarities between the equations (2.43) and (4.8), repeated here for convenience,

$$\begin{bmatrix} \hat{\underline{a}}(k+1|k) \\ \hat{\underline{b}}(k+1|k) \end{bmatrix} = \begin{bmatrix} R_{uu}(k) & -R_{uy}(k-1) \\ -R^T_{uy}(k-1) & R_{yy}(k) \end{bmatrix}^{-1} \begin{bmatrix} R_{uy}(k) \\ R_{yy}(k) \end{bmatrix} \quad (2.43a)$$

$$A_1 = B_1 C_1 \quad (2.43b)$$

$$\begin{bmatrix} R_{uu}(k) & R_{uy}(k-1) \\ R^T_{uy}(k-1) & R_{yy}(k) \end{bmatrix} \begin{bmatrix} \underline{a} \\ \underline{b} \end{bmatrix} = \begin{bmatrix} R_{du}(k) \\ R_{dy}(k) \end{bmatrix} \quad (4.8a)$$

$$\begin{bmatrix} \hat{\underline{a}} \\ \hat{\underline{b}} \end{bmatrix} = \begin{bmatrix} R_{uu}(k) & R_{uy}(k-1) \\ R^T_{uy}(k-1) & R_{yy}(k) \end{bmatrix}^{-1} \begin{bmatrix} R_{du}(k) \\ R_{dy}(k) \end{bmatrix} \quad (4.8b)$$

$$A_2 = B_2 C_2 \quad (4.8c)$$

The upper left partitions of  $B_1$  and  $B_2$  are obviously identical, in that the AKI and the Adaptive Recursive LMS both implement instantaneous estimates of the input covariance functions,  $R_{uu}(k)$ . The lower right partitions of  $B_1$  and  $B_2$  are similar with one subtle difference. Both are instantaneous covariance functions; however,  $B_1$  employs the covariance,  $R_{yy}(k)$ , of the actual data whereas  $B_2$  uses, in fact, previous filter estimates to compute the output covariance,  $R_{yy}^{(k)}$ , more properly denoted by  $\hat{R}_{yy}(k)$ . The negative sign of the upper right and lower left partitions in  $B_1$ , in comparison with  $B_2$ , is a result of the initial definition for the recursive

weights,  $b_i$ . That is, the transfer function for the general ARMA equation can be written as,

$$H_{\text{ARMA}}(z) = \frac{a_0 + \sum_{i=1}^m a_i z^{-i}}{1 - \sum_{i=1}^m b_i z^{-i}} \quad (4.9a)$$

or,

$$H'_{\text{ARMA}}(z) = \frac{a_0 + \sum_{i=1}^m a_i z^{-i}}{1 + \sum_{i=1}^n b_i z^{-i}} \quad (4.9b)$$

The second and more important observation is that cross-correlations employed by equation (4.8b) use the instantaneous values for the past estimates of the outputs while (2.43a) uses the actual past values for  $y(k)$ . Similarly, comparing  $C_1$  with  $C_2$ , one finds that the lower partition of  $C_2$  uses the cross-correlation between the desired signal and the past estimates of the adaptive recursive LMS filter.

Equation (4.8a) can convey the above information more clearly if it is instead written as,

$$\begin{bmatrix} \hat{a} \\ \cdots \\ \hat{b} \end{bmatrix} = \begin{bmatrix} R_{uu}(k) & R_{uy}^T(k-1) \\ \hline R_{uy}^T(k-1) & R_{yy}^T(k) \end{bmatrix}^{-1} \begin{bmatrix} R_{du}(k) \\ \cdots \\ R_{dy}^T(k) \end{bmatrix} \quad (4.10)$$

The convergence factors,  $k_a$  and  $k_b$ , can be compared to the steady state Adaptive Kalman Identifier gains by making

the identical analogies (3.8) as was done in the previous chapter. One also observes that since the output data that is processed by the Adaptive Recursive LMS filter are in actuality filter estimates, a more correct version of Feintuch's original algorithm (4.10) is written as,

$$\underline{a}(k+1) = \underline{a}(k) - 2k_a e(k) \underline{u}(k) \quad (4.11a)$$

$$\underline{b}(k+1) = \underline{b}(k) - 2k_b e(k) \hat{\underline{y}}(k) . \quad (4.11b)$$

In augmented form, equation (4.11) is,

$$\begin{aligned} \begin{bmatrix} \underline{a}(k+1) \\ \hline \underline{b}(k+1) \end{bmatrix} &= \begin{bmatrix} \underline{a}(k) \\ \hline \underline{b}(k) \end{bmatrix} - \begin{bmatrix} 2k_a \underline{u}(k) \\ \hline 2k_b \hat{\underline{y}}(k) \end{bmatrix} [d(k) - \underline{a}(k) \underline{u}(k) - \underline{b}(k) \hat{\underline{y}}(k)] \\ \begin{bmatrix} \underline{a}(k+1) \\ \hline \underline{b}(k+1) \end{bmatrix} &= \begin{bmatrix} \underline{a}(k) \\ \hline \underline{b}(k) \end{bmatrix} - \begin{bmatrix} 2k_a \underline{U}(k) \\ \hline 2k_b \hat{\underline{Y}}(k) \end{bmatrix} \left[ d(k) - [\underline{u}(k) \quad \hat{\underline{y}}(k)] \begin{bmatrix} \underline{a}(k) \\ \hline \underline{b}(k) \end{bmatrix} \right] . \end{aligned} \quad (4.12)$$

Recalling equation (2.15a) in a slightly different form we have,

$$\begin{bmatrix} \hat{\underline{a}}(k+1|k) \\ \hline \hat{\underline{b}}(k+1|k) \end{bmatrix} = \begin{bmatrix} \hat{\underline{a}}(k|k-1) \\ \hline \hat{\underline{b}}(k|k-1) \end{bmatrix} + K(k) \begin{bmatrix} z(k) - H(k) \\ \hline \end{bmatrix} \begin{bmatrix} \hat{\underline{a}}(k|k-1) \\ \hline \hat{\underline{b}}(k|k-1) \end{bmatrix} . \quad (4.13)$$

It is a simple matter to explore the similarities between (4.12) and (4.13) and make the following associations,

AKI

Adaptive Recursive LMS

$$\begin{bmatrix} \hat{\underline{a}}(k+1|k) \\ \hline \hat{\underline{b}}(k+1|k) \end{bmatrix} \Leftrightarrow \begin{bmatrix} \underline{a}(k+1) \\ \hline \underline{b}(k+1) \end{bmatrix} \quad (4.14a)$$

$$\begin{bmatrix} \hat{\underline{a}}(k|k-1) \\ \hline \hat{\underline{b}}(k|k-1) \end{bmatrix} \Leftrightarrow \begin{bmatrix} \underline{a}(k) \\ \hline \underline{b}(k) \end{bmatrix} \quad (4.14b)$$

$$z(k) \Leftrightarrow d(k) \quad (4.14c)$$

$$H(k) = [u(k) \dots u(k-m) \dots u(k-n)] \Leftrightarrow [\underline{u}(k) \underline{y}(k)] \quad (4.14d)$$

$$K(k) \Leftrightarrow \begin{bmatrix} 2k_a \underline{u}(k) \\ \hline 2k_b \underline{-}(k) \end{bmatrix} \quad (4.14e)$$

The associations (4.14d) and (4.14e) are the major differences between the AKI and the Adaptive Recursive LMS. It has been shown that the mean square error surface employing not only estimated feed forward coefficients but also estimated feedback (recursive) coefficients is in general multimodal [Ref. 36]. Hence, the Adaptive Recursive LMS algorithm does not minimize the mean square error [Ref. 19,20], and in general the gradient algorithm in this case does not seek the global minimum.

### C. OBSERVATIONS

The complex nature of estimating recursive coefficients is yet a formidable problem [Ref. 19,20]. It seems that Feintuch's algorithm is successful due to the a-priori knowledge of the minimal order generating the desired signal. This knowledge is used in setting the order of the adaptive recursive filter.

## V. SYNTHETIC DATA GENERATING PLANTS

The Adaptive ARMA Kalman Identifier was tested using computer derived data from several models. The input/output data was collected by driving a known plant with a zero mean, unit variance, white gaussian noise sequence of samples. The AKI algorithm was repeatedly tested using data from progressively more complex models.

### A. AUTOREGRESSIVE-MOVING AVERAGE DATA

Autoregressive-Moving Average (ARMA) data was generated using an equation of the form,

$$y(k) = b_1 y(k-1) + b_2 y(k-2) + \dots + b_n y(k-n) \\ + a_0 u(k) + \dots + a_m u(k-m) \quad (5.1)$$

Taking the Z-transform of (5.1) we have,

$$Y(z) = b_1 z^{-1} Y(z) + b_2 z^{-2} Y(z) + \dots + b_n z^{-n} Y(z) \\ + a_0 U(z) + \dots + a_m z^{-m} U(z) \quad (5.2)$$

where  $Y(z)$  ( $U(z)$ ) represents the Z-transform of the output (input) and  $z^{-n_0} Y(z) (z^{-n_0} U(z))$  represents the Z-transform of the output (input) delayed  $n_0$  time steps. Equation (5.1) has already been defined as the general form of the ARMA process provided that the sequence  $u(k-i)$   $i = 0, 1, \dots$  comes from a gaussian random process [Ref. 25]. The ratio,

$$H(z) = \frac{Y(z)}{U(z)} = \frac{a_0 + a_1 z^{-1} + \dots + a_m z^{-m}}{1 - b_1 z^{-1} - \dots - b_n z^{-n}} \quad (5.3)$$

can then be readily recognized as the general transfer function [Ref. 37] of a digital plant. That is, the  $u(k)$  is the observed input and the  $y(k)$  is the perfectly measured output. Consider that the measurements of the output include some random error,  $v(k)$ , due to the inaccuracy of the measuring instruments, then the resulting situation is as depicted in Figure 5.1. A practical, judicious and reasonable description of the measurement error is that this be a stochastic process whose distribution is zero mean gaussian. The noisy measurement,  $z(k)$ , is then the output,  $y(k)$ , plus the measurement error,  $v(k)$ .

There are three cases of interest: Case 1:  $b_i = 0$  for all  $i$ ; Case 2:  $a_i = 0$ ,  $i = 1, 2, \dots, m$ ; Case 3:  $a_i \neq 0$ ,  $b_i \neq 0$  for all  $i$ .

#### 1. Moving Average Data (Case 1)

Case one is readily recognized as the all zero plant which produces moving average data. A simple second order plant was defined where  $a_0 = 1.0$ ,  $a_1 = 2.0$  and  $a_2 = 3.0$ . The gaussian noise sequences,  $u(k)$  and  $v(k)$ , were obtained using the general purpose IMSL subroutine, GGNML, provided by the computer center at the Naval Postgraduate School.

#### 2. Autoregressive Data (Case 2)

Case two results in the all pole plant producing autoregressive data. The coefficients were selected such that the

plant of Figure 5.1 produced stable data. In precise terms, the poles of the function,

$$H(z) = \frac{1}{1 + \frac{26}{24} z^{-1} + \frac{9}{24} z^{-2} + \frac{1}{24} z^{-3}} \quad (5.4)$$

were located completely within the unit circle in the z-plane ensuring a stable plant [Ref. 37,38]. The noise sequences were obtained as before.

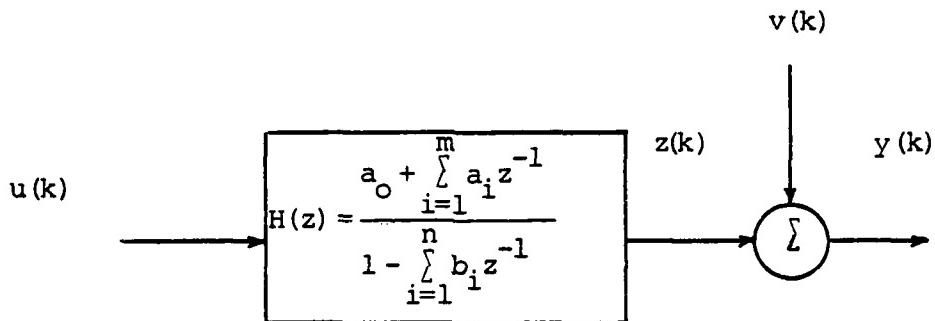


Figure 5.1. ARMA Digital Plant

### 3. Autoregressive-Moving Average Data (Case 3)

Case three follows from a logical combination of the two previous cases. That is, the transfer function,  $H(z)$ , now has both zeros and poles. In order to compare with previous work done in the area of system identification [Ref. 6], one of Perry's models [Ref. 6] was used. Namely, the transfer function  $H(z)$  used for this case was,

$$H(z) = \frac{1 + 1.4z^{-1} + .98z^{-2}}{1 - 1.14z^{-1} + 1.4549z^{-2} - .88490z^{-3} + .40745z^{-4}} \quad (5.5)$$

The input,  $u(k)$ , was obtained as in the previous two cases using the IMSL subroutine GGNML set to unit variance and zero mean. The output measurement noise sequence, however, was set to a variance of .0001. The basic software, program ARMA, which generated the different data is included as Appendix B. Basically, the coefficients A1-A10 change the character of the general ARMA equation (5.1) which can be selected to produce the desired plant data. The input/output data is then written onto a disk file for subsequent analysis.

In order to compare some of the findings in this thesis with the results previously obtained by Feintuch [Ref. 18], the transfer function,

$$H(z) = \frac{0.05 - 0.40z^{-1}}{1.0 - 1.1314z^{-1} + 0.25z^{-2}} \quad (5.6)$$

was also used as a source of synthetic data. Feintuch used (5.6) as a source of data with which to analyze the operation of his Adaptive Recursive LMS Filter.

#### B. PHASE LOCKED LOOP DATA

The second class of data used to exercise the Adaptive ARMA Kalman Identifier was derived from a computer simulated phase locked loop (PLL) developed by Romeo [Ref. 7]. The basic PLL algorithm, however, implemented a forward Euler integration scheme.

Briefly, in block diagram form, the phase tracking characteristics of the PLL in the frequency domain can be depicted as in Figure 5.2.

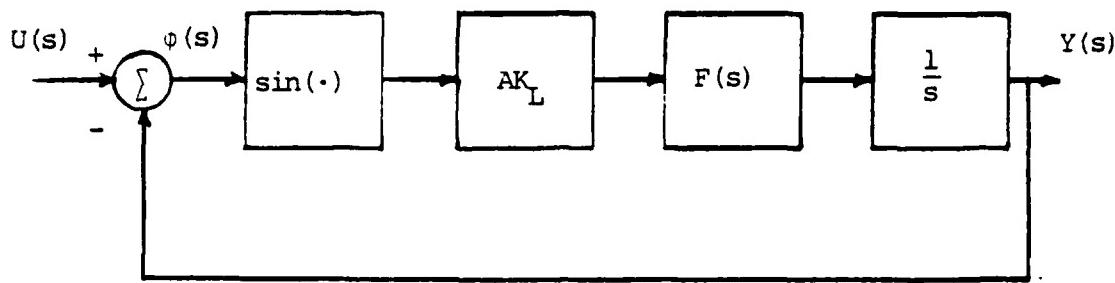


Figure 5.2. PLL Block Diagram

Where:

A is the input voltage

K is the loop gain

$F(s)$  is the filter transfer function

For most applications, it is assumed that  $\phi(t) \ll \pi/2$  allowing one to make the approximation,  $\sin \alpha \approx \alpha$ . This is the linear mode of operation. Romeo [Ref. 7] chose the filter characteristics  $F(s) = 1 + K/S$  and the same characteristics were used in this thesis in order to compare results. The parameters of the overall system were adjusted to obtain a damping coefficient,  $\zeta$ , of 0.3 and a natural frequency,  $w_n$  of 3.33 rad/sec. This resulted in a step response overshoot of about 46.7% at  $t \approx .75$  sec. Solving for the time domain step response analytically in the linear region one obtains,

$$y(t) = 1 + 1.048e^{-t} \sin(3.178t - 1.266) \quad (5.7)$$

The final block diagram of the PLL system is shown in Figure 5.3.

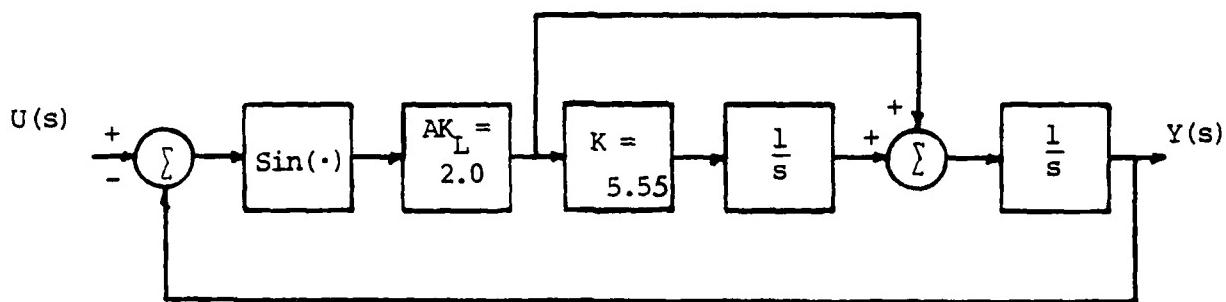


Figure 5.3. Final PLL Block Diagram

Normalized step responses for several step magnitudes using Digital Simulation Language (DSL) were obtained. Figure 5.4 shows the step responses of several step inputs in the range  $0 < u(t) < 30$ . It is obvious that the PLL is in its approximate linear region and does not exhibit any discernible distortion. Figure 5.5 is a repeat of the above test; however, the range of step inputs are  $30 \leq u(t) \leq 170$ . It is apparent from Figure 5.5 that the  $\sin (\cdot)$  nonlinearity begins to have some effect on the operation of the PLL in that the amplitude of the responses are reduced and delayed.

The DSL PLL system, however, was not used as a source of synthetic data because of the problem of nonstationary statistics of the input signal using the random sequence generators available under DSL already discussed in Ref. 7. These simulations were nevertheless used as a basis for comparison of the operation of the discrete PLL developed in Reference 7 but modified to implement a forward Euler integration scheme which is used in this thesis.

2ND ORDER PLL -- TRAPEZOIDAL INTEGRATION  
NORMALIZED STEP RESPONSE .001/.01/.1/1/10/20/30

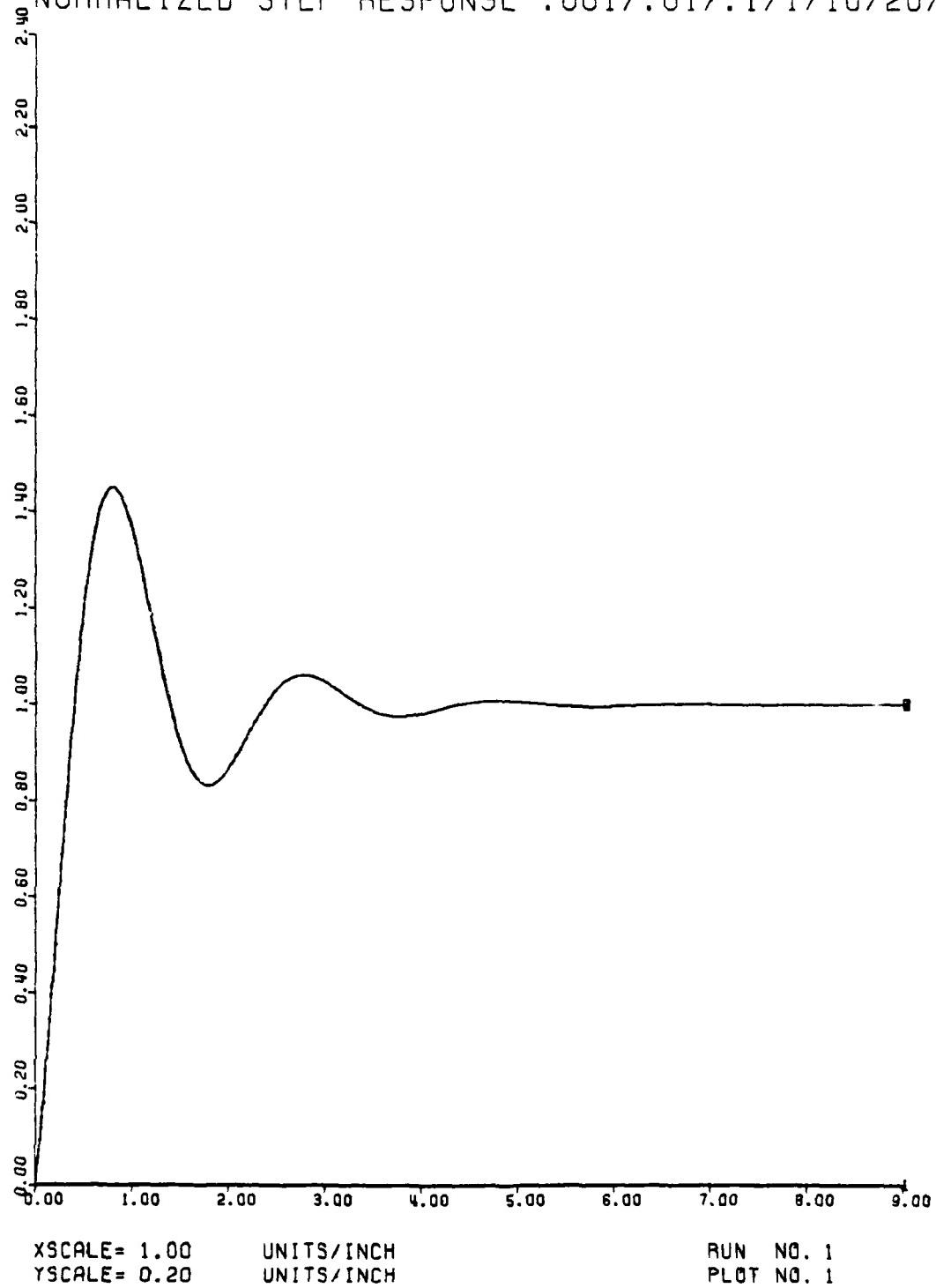


Figure 5.4. PLL Step Response, Linear Region

2ND ORDER PLL -- TRAPEZOIDAL INTEGRATION  
NORMALIZED STEP RESPONSE 1/30/60/90/120/150/170

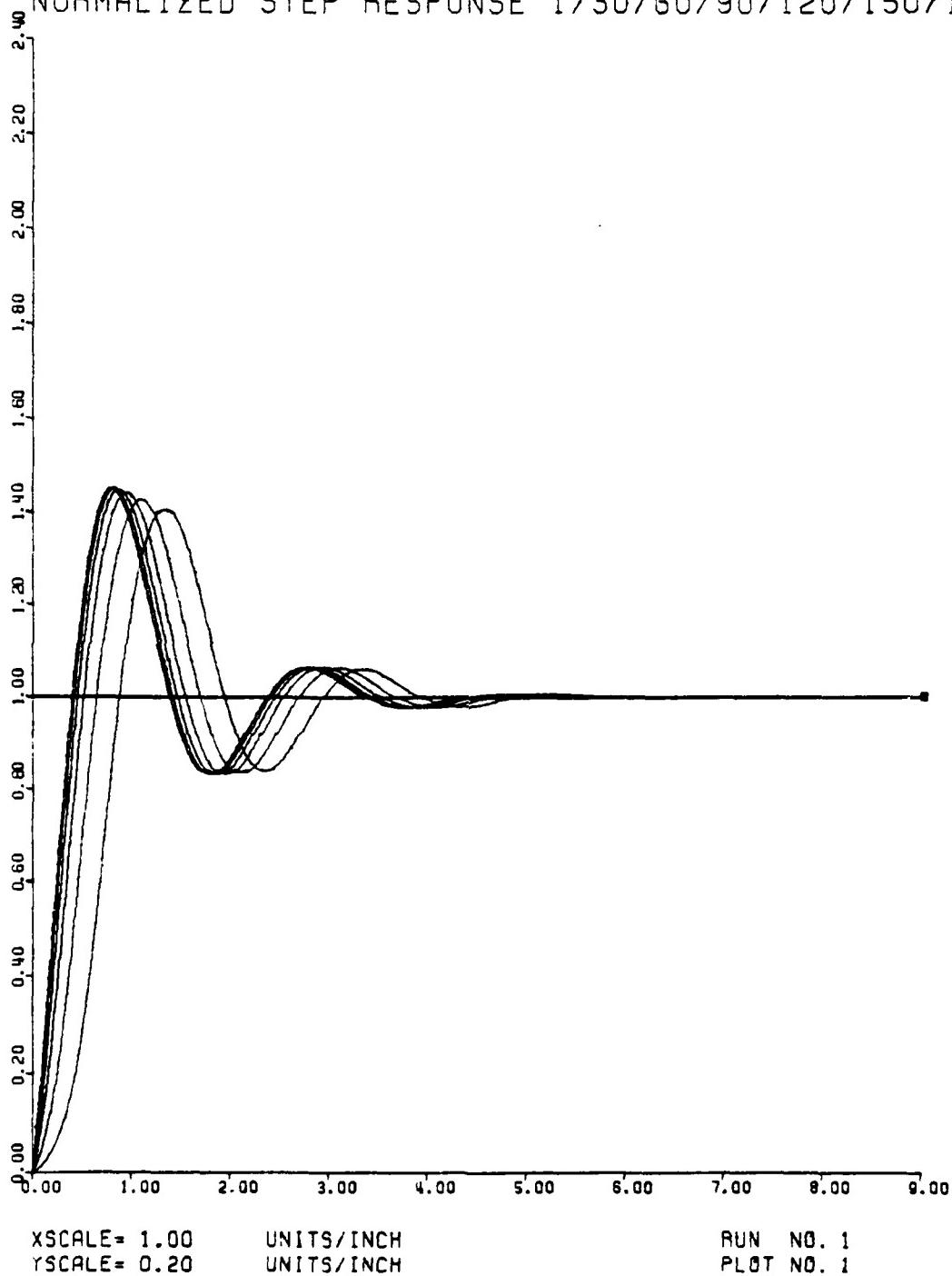


Figure 5.5. PLL Step Response, Non-Linear Region

The forward Euler integrator takes the form,

$$\frac{1}{s} \iff \frac{Tz^{-1}}{1 - z^{-1}}, \quad T = .01 \quad (5.8)$$

The block diagram of Figure 5.3 changes character to appear as in Figure 5.6.

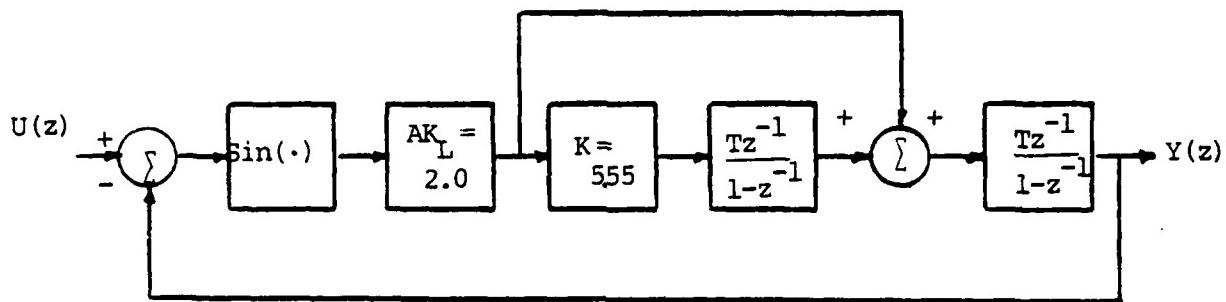


Figure 5.6. Discrete PLL Block Diagram

Applying Mason's gain rule to the block diagram of Figure 5.6 in the linear region we arrive at,

$$\frac{Y(z)}{U(z)} = H(z) = \frac{0.02z^{-1} - 0.018890z^{-2}}{1 - 1.980z^{-1} + .981110z^{-2}} \quad (5.9)$$

the linearized PLL transfer function. The discrete PLL system has zeros at  $(0.0, .9445)$  and a complex conjugate pole pair at  $(0.990 \pm j.03178)$ .

The discrete PLL system was implemented using a Fortran program on the IBM 370 in double precision. The source code for the discrete PLL is included as Appendix C. The discrete

PLL was tested using various magnitude step inputs. Results were similar to those from the DSL simulation. That is, the discrete PLL exhibited the same approximate overshoot, rise time, natural frequency and nearly identical time of max overshoot in the linear region. To implement the PLL using forward Euler integration, the block diagram of Figure 5.6 was reconfigured to appear as Figure 5.7. Note that Figure 5.7 does not employ any delay free loops, hence it is easily programmable.

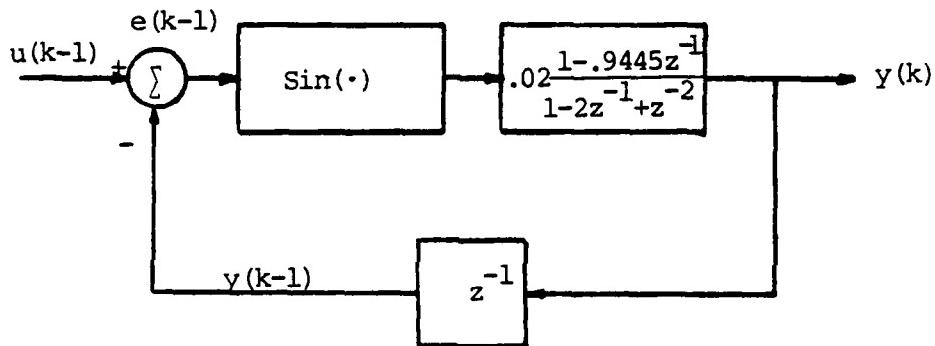


Figure 5.7. Programmable PLL Implementing Forward Euler Integration

Figure 5.8 summarizes the results of the discrete PLL simulation for normalized step inputs in the range  $0 < u(t) \leq 30$ . Similarly, the discrete PLL was tested in the nonlinear region using normalized step inputs in the range  $30 \leq u(t) \leq 170$ . The discrete PLL displayed the same characteristics of decreased output amplitude and delay which was first observed in the DSL simulation. Figure 5.9 summarizes the test results of the discrete PLL in the nonlinear region.

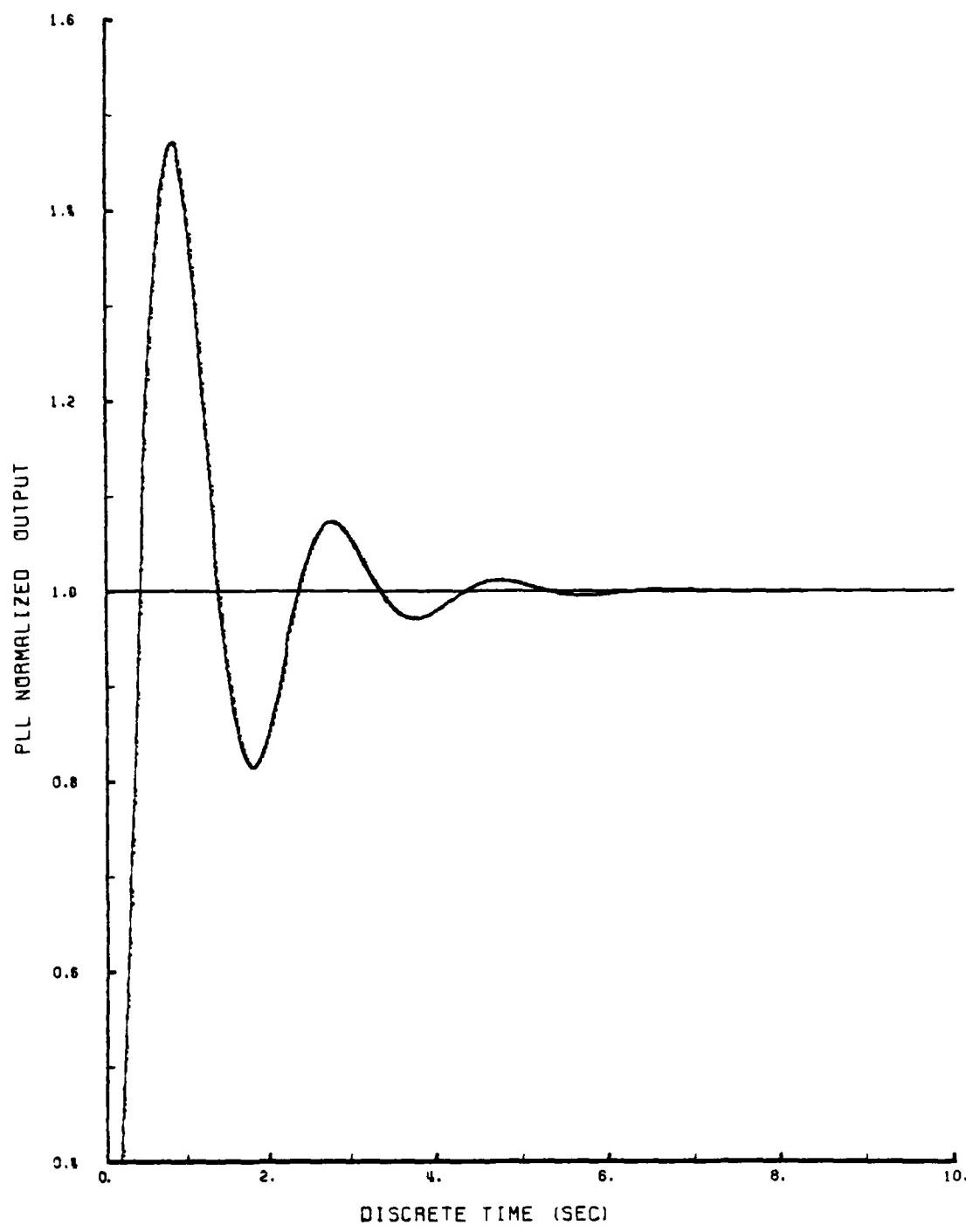


Figure 5.8. Discrete PLL Step Response, Linear Region

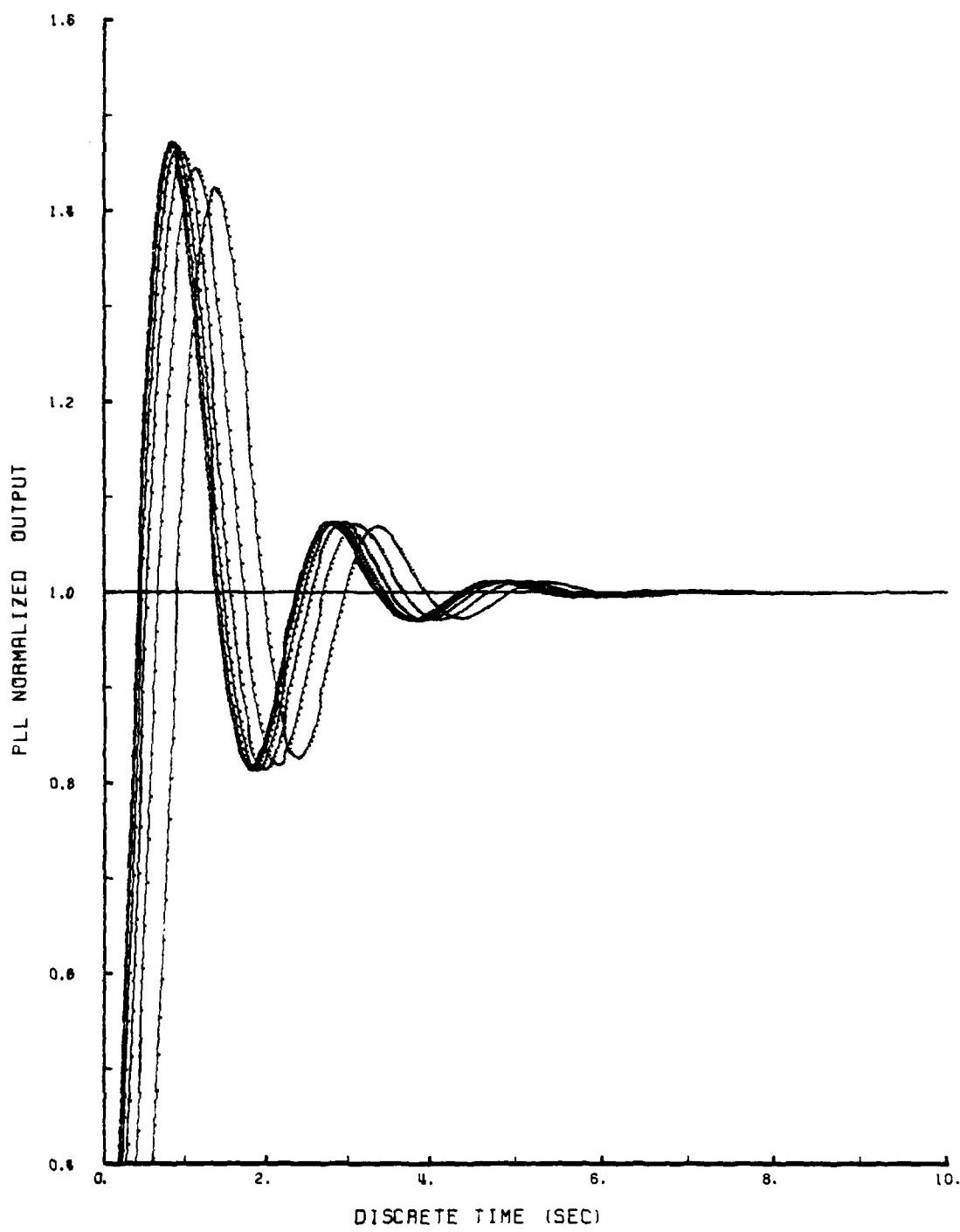


Figure 5.9. Discrete PLL Step Response, Nonlinear Region

It was felt that the PLL was adequately modeled for use as a source of data to be subsequently analyzed by the Adaptive Kalman Identifier.

## VI. MODEL IDENTIFICATION SOFTWARE

Three basic Fortran IV programs for use on the IBM 370 were developed:

- (1) ADAPTSN: Adaptive Kalman Identifier
- (2) LMS: Adaptive Least Mean Square Filter
- (3) LMSR: Adaptive Recursive Least Mean Square Filter

All three programs used double precision arithmetic to minimize the effects of truncation error, limit cycles and roundoff error. In all cases the orders of the autoregressive and moving average processes were read in unformatted form from a disk file along with the input/output data of the unknown system to be analyzed. An attempt was made to minimize memory storage, but not at the expense of program flow and clarity.

### A. ADAPTIVE KALMAN IDENTIFIER

Program ADAPTSN is a versatile Fortran IV software program which implements equations (2.15) to identify the coefficients of an MA, AR, or ARMA process. Additionally, at each iteration the poles and zeros of the evolving transfer function are computed. ADAPTSN can also be used to perform regression analysis on non-linear terms as discussed in Chapter VII. The versatility of the AKI lies in the various options which are available to the user. By properly selecting the flags N, M, and NL, the user can perform regression analysis on data whose combined order ( $N+M+1$ ) is less than or equal to 20. Table 6.1 lists the ascribed meanings of the various flags.

Table 6.1  
AKI FLAG MEANINGS

- N: the order of the autoregressive process
- M: one greater than the order of the moving average process
- NL = 0: AKI is used to identify the coefficients associated with the general ARMA equation (2.1b) (linear regression)
- NL = 1: AKI is used to identify the coefficients of the linear general ARMA equation and the weighting coefficient associated with  $u^3(k-i)$
- NL = 2: AKI is used as in NL = 0,1 and additionally identifies the weighting coefficient associated with  $y^3(k-i)$
- NL = 3: AKI is used as in NL = 0,1,2 and additionally identifies the weighting coefficient associated with  $u^2(k-i)y(k-i)$
- NL = 4: AKI is used as in NL = 0,1,2,3, and additionally identifies the weighting coefficient associated with  $y^2(i-i)u(k-i)$
- NL = 5: AKI is implemented to analyze time series and identify the ARMA coefficients associated with the Box-Jenkins model [Ref. 26].

The option given by NL = 5 was not extensively tested and as such only limited results are available. Table 6.2 outlines the allowable flag combinations which will produce a valid analysis of the data.

For purposes of this thesis, the non-linear (NL) options were configured to implement the expansion terms associated with the Taylor series expansion of the sine function. This, however, can be changed at the user's discretion to implement any other series expansion by inserting the appropriate Fortran statements at the proper location in the AKI program.

Table 6.2  
VALID FLAG COMBINATIONS

OPTION (NL)	MAX COMBINED ORDER
0	$N + M \leq 20$
1	$N + 2M \leq 20; N, M \neq 0$
2	$2N + 2M \leq 20; N, M \neq 0$
3	$3N + 2M \leq 20; N, M \neq 0$
4	$4N + 2M \leq 20; N, M \neq 0$
5	$N + M \leq 20$

The overall structure of the AKI program uses subroutine calls to compute the various quantities necessary for the eventual computation of the system ARMA coefficients. These subroutines are: GAIN, RECKON, PRINT, NEXT. A brief description of the function each performs is given at the beginning of each subroutine. Figure 6.1 describes the general program flow of the AKI. The source code for program ADAPTSN is included as Appendix D.

#### B. ADAPTIVE LEAST MEAN SQUARE FILTER

Program LMS uses the equations developed in Chapter III to compute the coefficients (or weights) associated with a moving average process. The LMS filter program is capable of computing up to 12 MA coefficients; or equivalently, is capable of identifying an eleventh order moving average process. The general program flow of the LMS filter is presented in Figure 6.2 and the annotated source code is included as Appendix E.

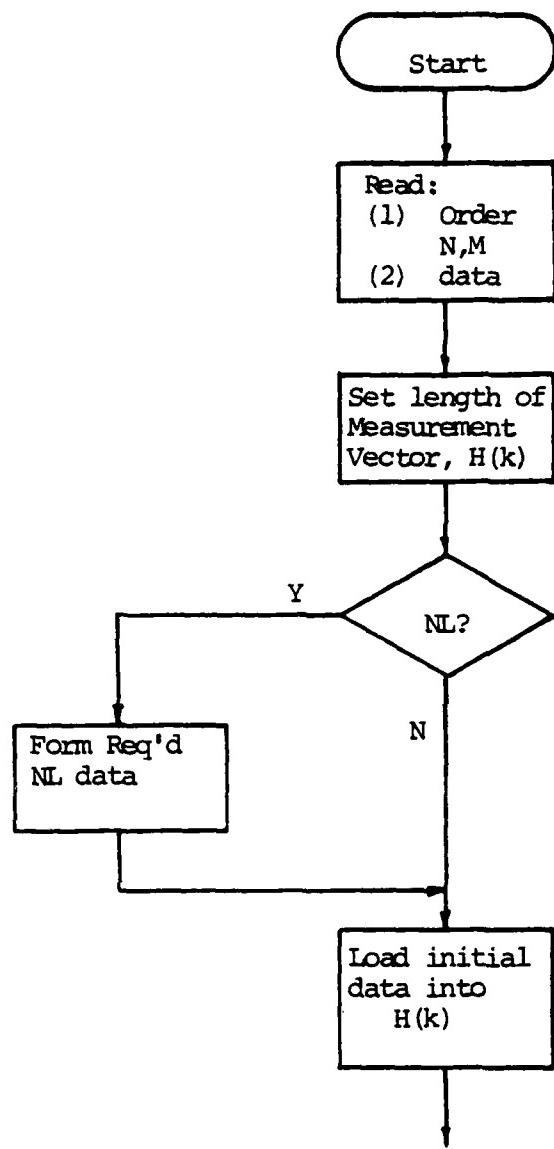


Figure 6.1. AKI General Program Flow

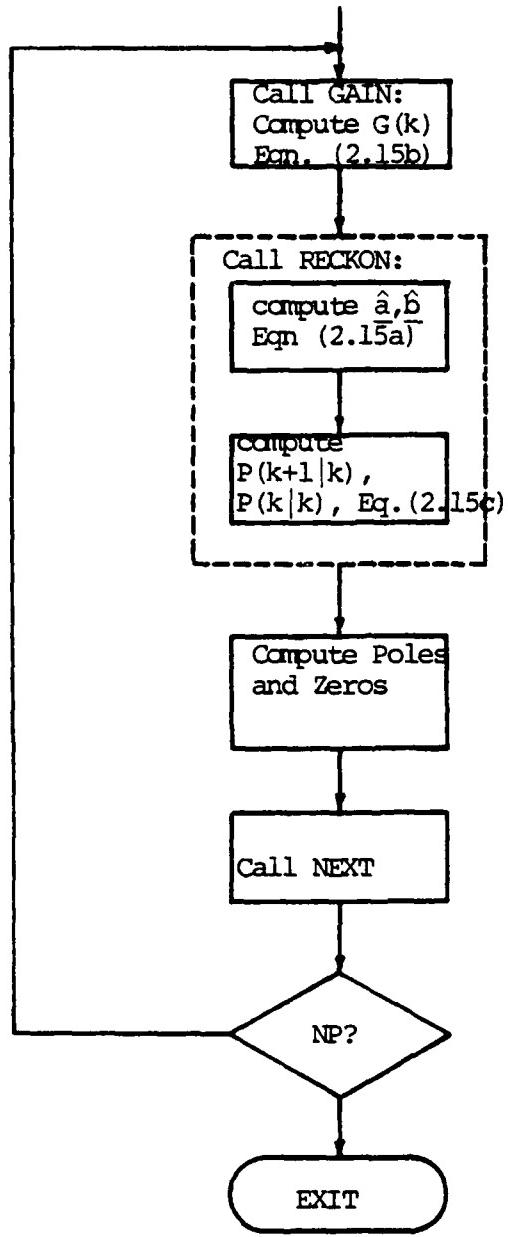


Figure 6.1. (CONTINUED)

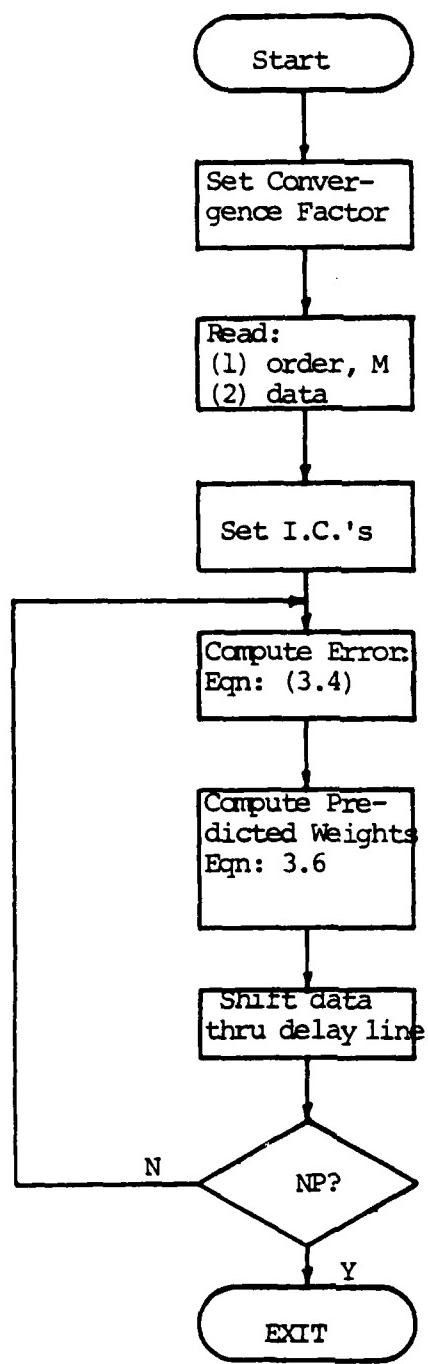


Figure 6.2. LMS Filter Program Flow

### C. ADAPTIVE RECURSIVE LEAST MEAN SQUARE FILTER

Program LMSR realizes the algorithm proposed by Feintuch and recapped in Chapter IV. It was designed to handle a combined AR and MA order of 11. That is, the program can be used to estimate  $m + 1 = M$  coefficients associated with the MA process and  $n = N$  coefficients associated with the AR process such that  $N + M \leq 12$ . The general program flow for the Adaptive Recursive LMS filter is shown in Figure 6.3 and the source code is included as Appendix F.

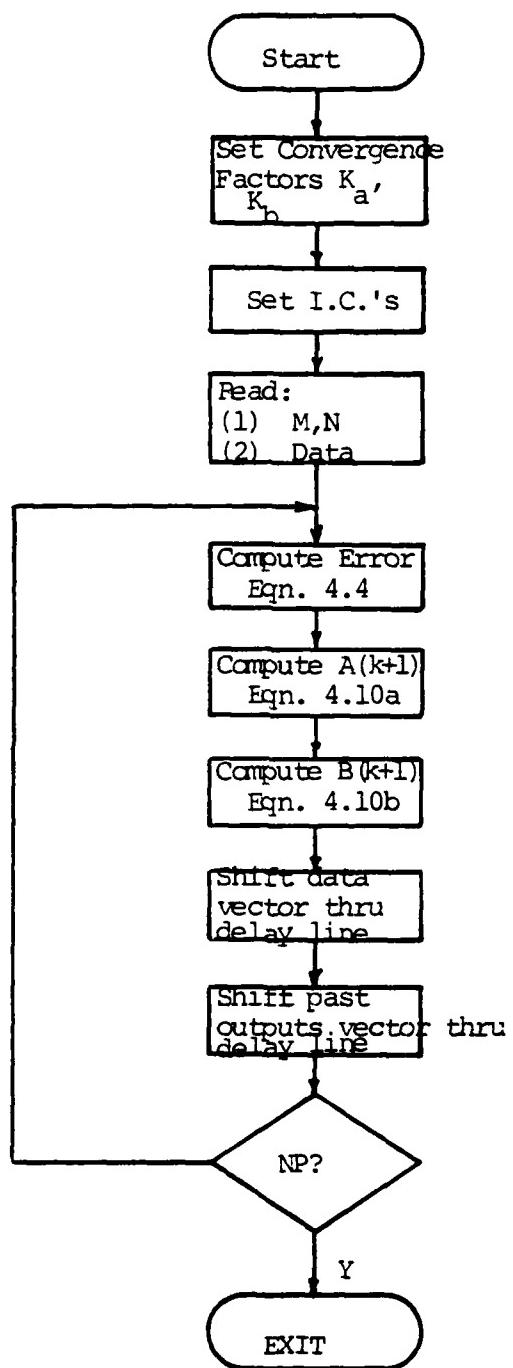


Figure 6.3. LMSR Filter Program Flow

## VII. NON-LINEAR IDENTIFICATION

The Adaptive Kalman Identifier can be modified to estimate the weighting coefficients associated with higher order terms produced by some non-linearity within the unknown system. In general, much has to be known about the non-linearity such that the functional description chosen for it is a close approximation to the effects it causes. In this thesis one possible application of the Adaptive Kalman Identifier toward identifying a system with a known non-linearity is explored. No attempt has been made to present an extensive treatment of non-linear analysis techniques.

The approach taken has been previously explored by Parker [Ref. 8]. A special case of the generalized non-linear ARMA model [Ref. 8],

$$\begin{aligned}
 y(k) = & \sum_{i_1=0}^{\infty} a(i_1)u(k-i_1) + \sum_{i_1=0}^{\infty} \sum_{i_2=0}^{\infty} a(i_1, i_2)u(k-i_1)u(k-i_2) + \dots \\
 & + \sum_{i_1=0}^{\infty} \dots \sum_{i_m=0}^{\infty} a(i_1, \dots, i_m)u(k-i_1) \dots u(k-i_m) \\
 & + \sum_{j_1=1}^{\infty} b(j_1)y(k-j_1) + \sum_{j_1=1}^{\infty} \sum_{j_2=1}^{\infty} b(j_1, j_2)y(k-j_1)y(k-j_2) + \dots \\
 & + \sum_{j_1=1}^{\infty} \dots \sum_{j_n=1}^{\infty} b(j_1, j_2, \dots, j_n)y(k-j_1)y(k-j_2) \dots y(k-j_n) \\
 & + \sum_{i_j=0}^{\infty} \sum_{j_1=1}^{\infty} c(i_j, j_1)u(k-i_j)y(k-j_1) + \dots + \sum_{i_1=0}^{\infty} \dots \sum_{i_m=0}^{\infty} \sum_{j_1=1}^{\infty} \\
 & \dots \sum_{j_n=1}^{\infty} c(i_1, \dots, i_m, j_1, \dots, j_n)u(k-i_1)\dots u(k-i_m)y(k-j_1)\dots y(k-j_n)
 \end{aligned} \tag{7.1}$$

(7.1)

is used to model the input/output relationships, non-linear transfer function, of the phase locked loop. The non-linear element, the sine function, of the PLL can be replaced by a Taylor power series expansion,

$$\sin(x) = x - \frac{1}{3!} x^3 + \frac{1}{5!} x^5 - \dots \quad (7.2)$$

Other expansions can be used, e.g., Legendre polynomials, Volterra series, ..., etc.; however, only the Taylor expansion was investigated in this thesis. Practical implementation of (7.2) suggests truncation at the third order term. The sine function is therefore approximated as

$$\sin(x) = x - \frac{1}{3!} x^3 . \quad (7.3)$$

Substituting (7.3) into the block diagram of Figure (5.6) we have Figure 7.1.

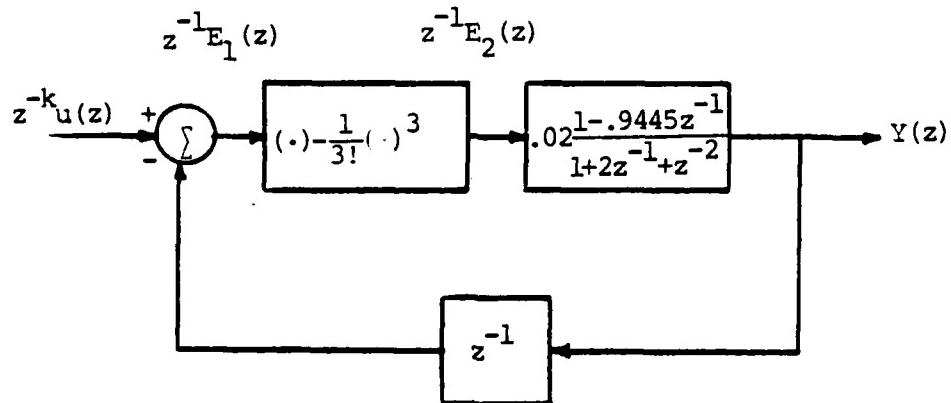


Figure 7.1. PLL, Third Order Taylor Series Approximation

The discrete time domain equations at the different nodes are:

$$e_1(k-1) = u(k-1) - y(k-1) \quad (7.4a)$$

$$e_2(k-1) = e_1(k-1) - \frac{1}{3!} e_1^3(k-1) \quad (7.4b)$$

$$\begin{aligned} y(k) = & .02e_2(k-1) - .01889e_2(k-2) + 2.0y(k-1) \\ & - y(k-2) \end{aligned} \quad (7.4c)$$

Manipulating equations (7.4a)-(7.4c) gives

$$\begin{aligned} y(k) = & .02u(k-1) - .01889u(k-2) + 1.98y(k-1) - .98111y(k-2) \\ & - .003333u^3(k-1) + .003148333u^3(k-2) \\ & + .003333y^3(k-1) - .003148333y^3(k-2) \\ & + .01u^2(k-1)y(k-1) - .09445u^2(k-2)y(k-2) \\ & - .01u(k-1)y^2(k-1) + .09445u(k-2)y^2(k-2). \end{aligned} \quad (7.5)$$

Equation (7.5) indicates which non-linear terms of equation (7.1) should be retained. Therefore, the third order non-linear approximation model should contain the following terms:

$$u(k-1), u(k-2), y(k-1), y(k-2) \quad \text{linear terms} \quad (7.6a)$$

$$u^3(k-1), u^3(k-2) \quad \text{input cubic terms} \quad (7.6b)$$

$$y^3(k-1), y^3(k-2) \quad \text{output cubic terms} \quad (7.6c)$$

$$\left. \begin{array}{l} u^2(k-1)y(k-1), u^2(k-2)y(k-2) \\ y^2(k-1)u(k-1), y^2(k-2)u(k-2) \end{array} \right\} \text{third order cross product terms} \quad (7.6d)$$

The AKI algorithm is primarily modified to include the hybrid signals (7.6) in the structure of  $H(k)$ , the measurement vector. Since now the measurement vector,  $H(k)$  takes the form,

$$\begin{aligned} H(k) = & [u(k) \dots u(k-m), -y(k-1) \dots -y(k-n), u^3(k) \dots \\ & \dots u^3(k-m), -y^3(k-1) \dots -y^3(k-n), -(u^2(k-1)y(k-1)) \dots \\ & \dots -(u^2(k-n)y(k-n)), (u(k-1)y^2(k-1)) \dots (u(k-n)y^2(k-n))] \end{aligned} \quad (7.7)$$

the AKI algorithm calculates the coefficients associated with the special case of the generalized non-linear ARMA model, equation (7.1).

Romeo using a similar approach [Ref. 7] computes a least squares curve fit for the third order truncation of the Taylor series for the sine function over the interval  $(0, \pi/2)$ . Beginning with the approximation,

$$\sin(x) \approx \alpha x + \beta x^3 \quad (7.8)$$

Romeo finds the values for  $\alpha$  and  $\beta$  to be .865 and -.095 respectively. Performing the same operations as those used in obtaining equation (7.5) we arrive at,

$$\begin{aligned}
y(k) = & .02\alpha u(k-1) - .01889\alpha u(k-2) + (2.0 - .02\alpha)y(k-1) \\
& + (.01889\alpha - 1)y(k-2) - .02\beta u^3(k-1) + .01889\beta u^3(k-2) \\
& - .01889\beta y^3(k-2) + .06\beta u^2(k-1)y(k-1) + .02\beta y^3(k-1) \\
& - .056670\beta u^2(k-2)y(k-2) - .06\beta y^2(k-1)u(k-1) \\
& + .056670\beta y^2(k-2)u(k-2)
\end{aligned} \tag{7.9}$$

Table 7.1 tabulates the resulting weighting coefficients using the three methods just described: (1) analytically calculated values using the truncated Taylor series, (2) least squares estimate of the third order sine approximation, as computed by Romeo [Ref. 7] and (3) the coefficient estimates as computed using the AKI algorithm.

The tabulation clearly shows that the AKI outperforms the first two methods overall. That is, the linear terms were identified without question; however, the AKI failed to identify the coefficients associated with the  $y^3(i-1)$ ,  $y^3(k-2)$ ,  $u^2(k-2)$   $y(k-2)$  and  $y^2(k-2)$   $u(k-2)$  terms. The reason for the failure is not known and was not investigated.

Table 7.1  
NON-LINEAR WEIGHTING COEFFICIENTS CALCULATED  
USING THREE METHODS

(1)	(2)	(3)
TAYLOR SERIES	LEAST SQUARES EST.	AKI USING ACTUAL
$\sin(x) = x - \frac{1}{3!}x^3$	$\sin(x) = \alpha x + \beta x^3$ $\alpha = .865, \beta = -.095$	$\sin(x)$ data base
y(k-1)	1.98	1.980
y(k-2)	-.98111	-.9815
u(k)	0.0	.00003691
u(k-1)	0.2	.02003
u(k-2)	-.01889	-.01892
$u^3(k)$	0.0	.00001471
$u^3(k-1)$	-.003333	-.003278
$u^3(k-2)$	.003148333	.003032
$y^3(k-1)$	.003333	-.05695
$y^3(k-2)$	-.003148333	.05799
$u(k-1)y(k-1)$	.01	.01032
$u^2(k-2)y(k-2)$	-.09445	-.009610
$y^2(k-1)y(k-1)$	-.01	-.01640
$y^2(k-2)u(k-2)$	.09445	.01913

## VIII. FINDINGS AND CONCLUSION

The performance of the Adaptive Kalman Identifier was compared to the LMS Adaptive Filter and the Adaptive Recursive LMS Filter using data derived from the models discussed in Chapter V. Results for the case where the order of the model (that is, m and n of equation (2.1b)) are assumed known are presented first followed by the analysis of the PLL data. The findings for the overmodeled case are presented next. The graphs show typical runs and do not represent ensemble averages.

### A. ORDER OF THE UNKNOWN SYSTEM IS KNOWN

#### 1. AKI vs. LMS Adaptive Filter

Using the MA form of the Adaptive Kalman Identifier, its performance can be compared against that of the LMS adaptive filter. Synthetic data derived from a plant whose transfer function is,

$$H(z) = 1.0 + 2.0z^{-1} + 3.0z^{-2} \quad (8.1)$$

was used. However, a fair comparison necessitated that the LMS Adaptive filter be "tuned" by adjustment of the convergence factor,  $k_s$ . Several values for  $k_s$  in the range [-.600, -.200] were used. The objective of tuning the LMS filter was to achieve a fast convergence time with little or no steady state error while not compromising filter stability. The three filter weights were normalized and plotted for five

convergence factor values. It can be seen from Figures 8.1-8.3 that convergence is essentially reached by step thirty-five, ( $k \approx 35$ ). As the convergence factor is further increased it can be noted in Figure 8.4 that the filter weights become more noisy, and that when  $k_s = -.600$ , filter stability is being compromised (Figure 8.5a).

Using the same data, the AKI converges to the MA coefficients in less than five iterations. Referring to Figure 8.5b, it was also noted that as the measurement noise  $v(k)$  was increased, the LMS algorithm yielded more noisy estimates whereas the AKI tended to compensate for measurement noise. This is intuitively reasonable since the AKI incorporates into its algorithm the effects of measurement error due to noisy sensors. Comparing Figure 8.6 with Figure 8.7 the latter figure clearly shows the faster convergence of the AKI to the plant coefficient.

It was shown in Chapter III that the AKI gains would approach a steady state value and indeed they did, as Figure 8.8 shows. However, the instantaneous Kalman gains display a more erratic pattern as shown in Figure 8.9. The information of Figure 8.8 was gleaned from Figure 8.9 by computing the average of the individual gains at each iteration using the algorithm,

$$E\{\underline{G}(k+1)\} = E\{\underline{G}(k)\} + \frac{1}{k+1} [\underline{G}(k) - E\{\underline{G}(k)\}] \quad (8.2)$$

where,

$E\{\underline{G}(k+1)\}$  is the one step prediction of the average value for the gains,

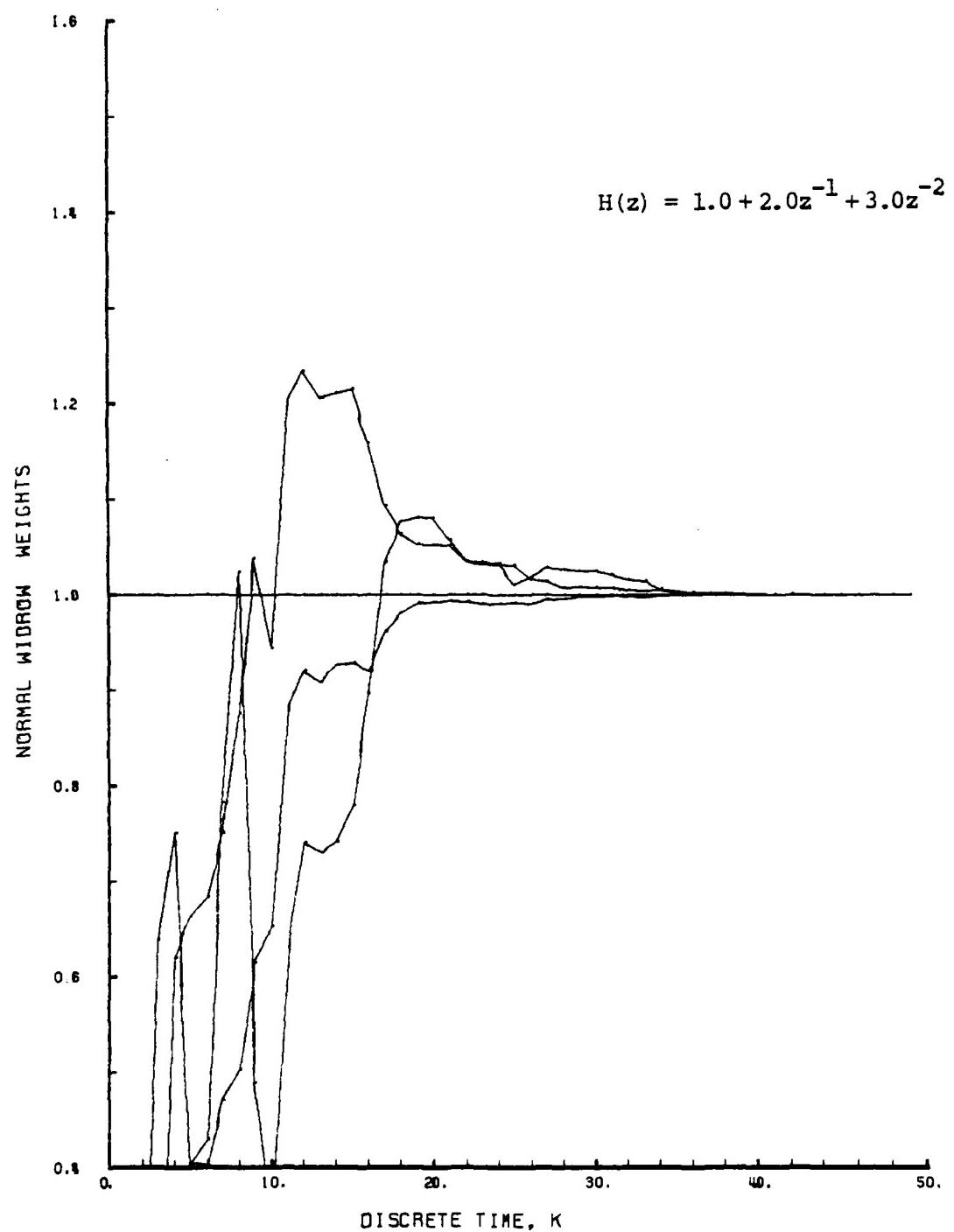


Figure 8.1. LMS Adaptive Filter Weights,  $k_s = -.200$

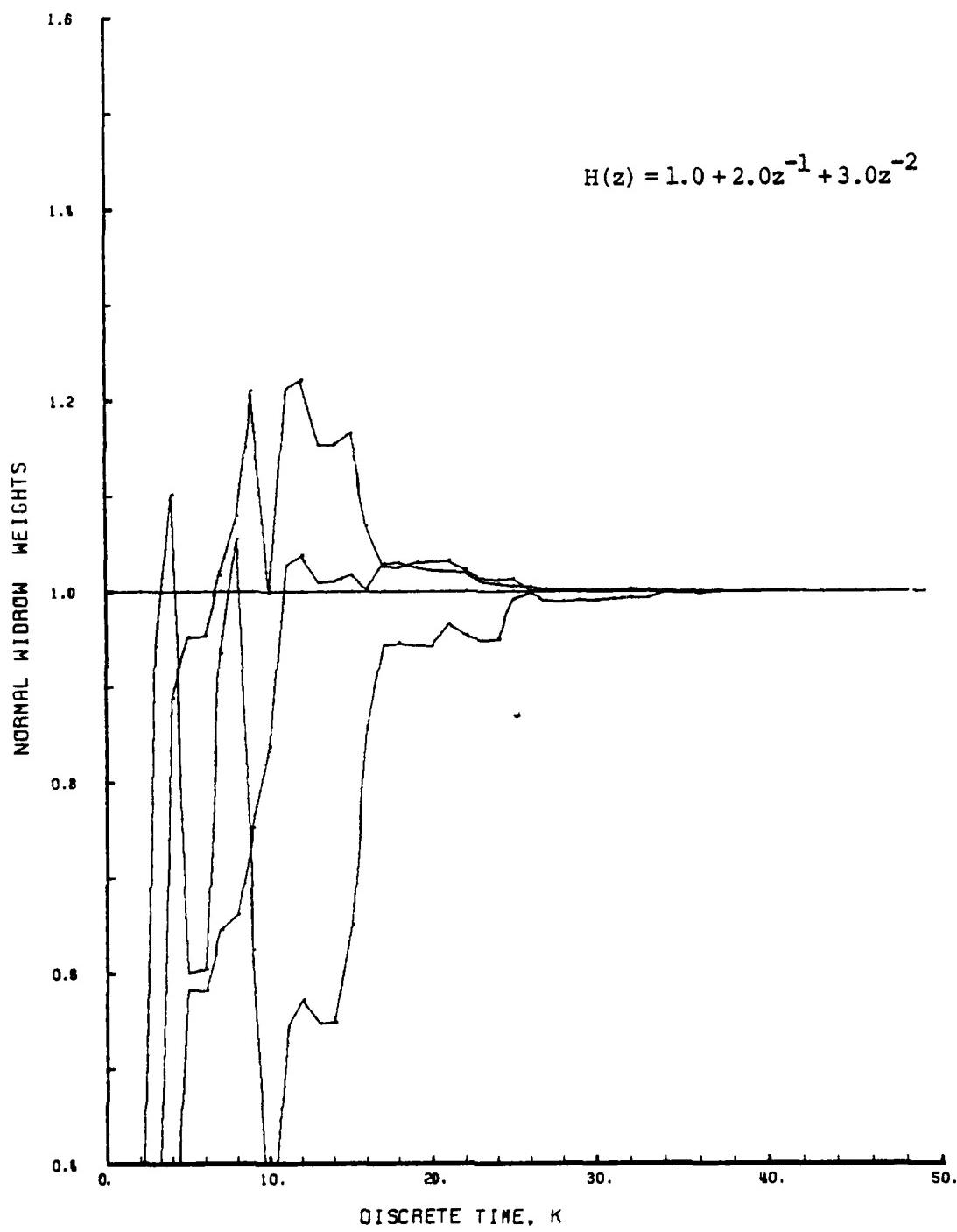


Figure 8.2. LMS Adaptive Filter Weights,  $k_s = -.300$

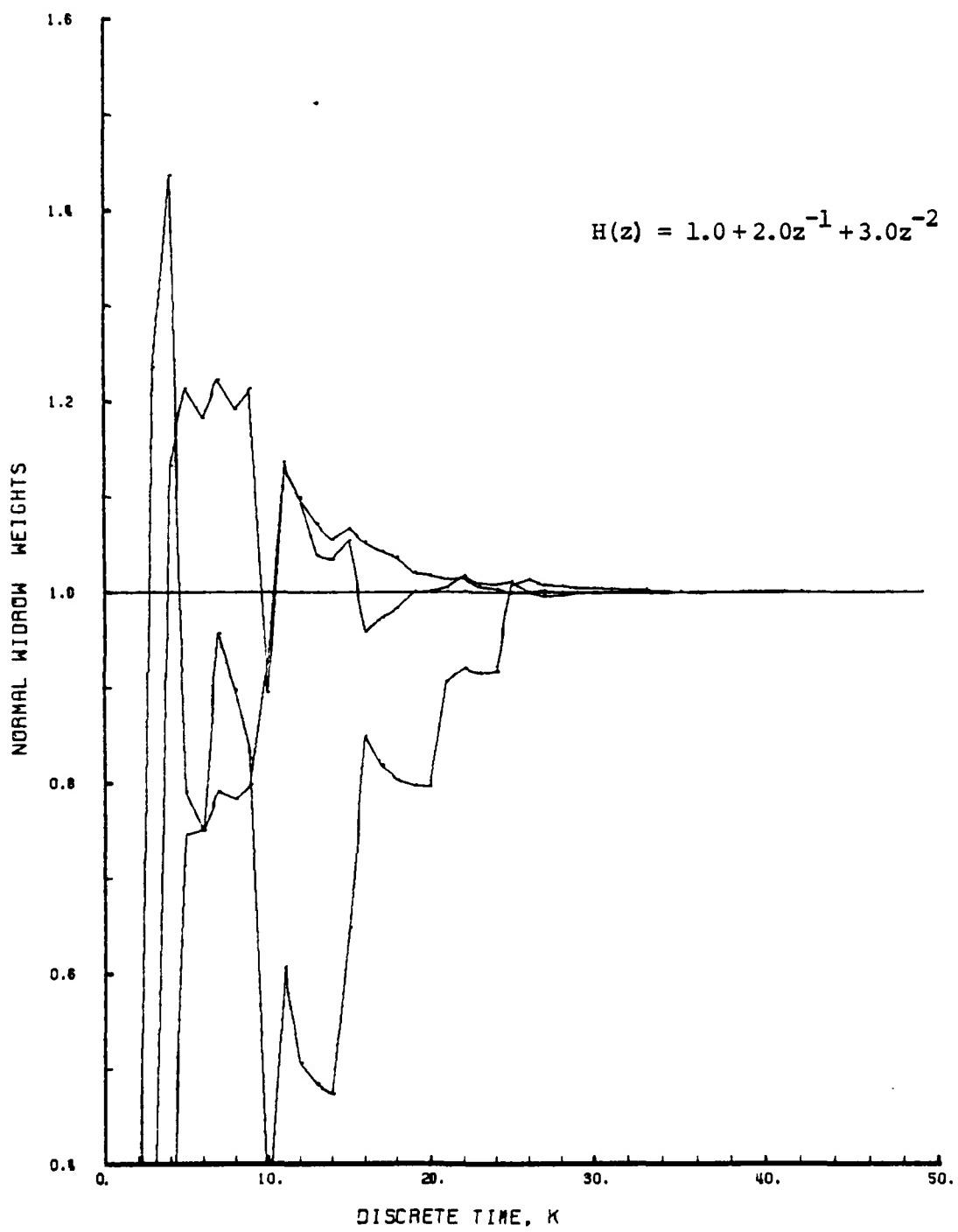


Figure 8.3. LMS Adaptive Filter Weights,  $k_s = -.400$

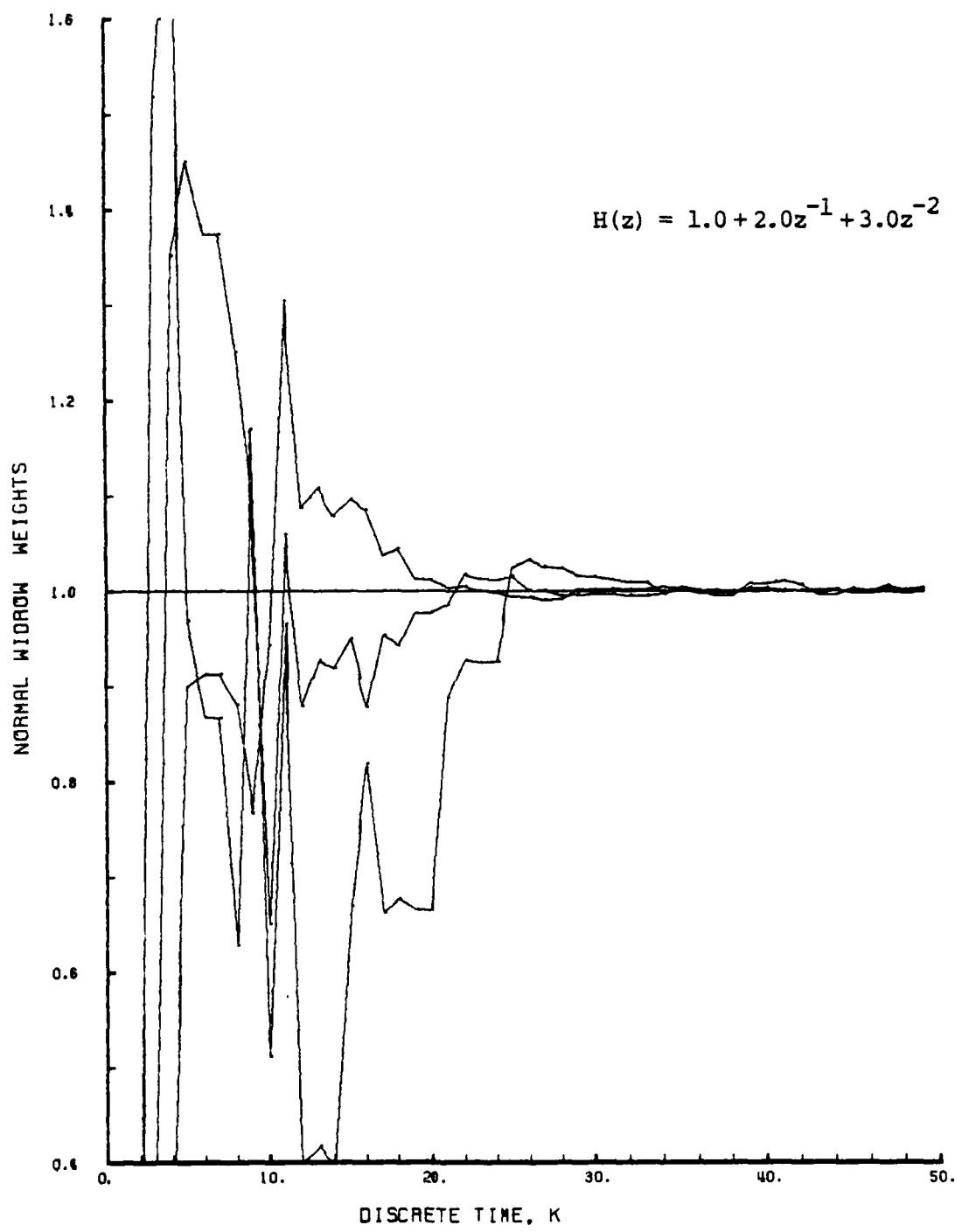


Figure 8.4. LMS Adaptive Filter Weights,  $k_s = -0.500$

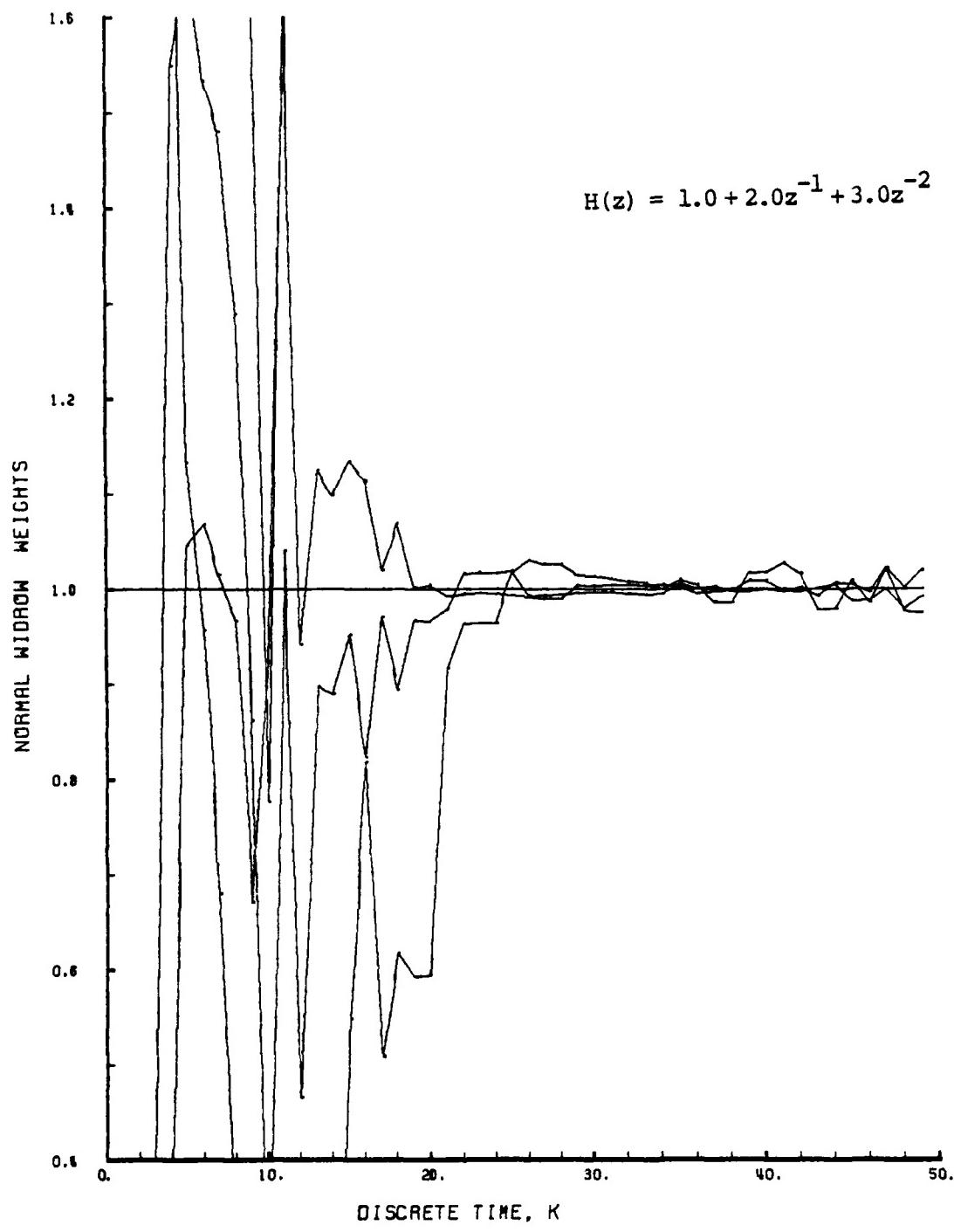


Figure 8.5a. LMS Adaptive Filter Weights,  $k_s = -.600$

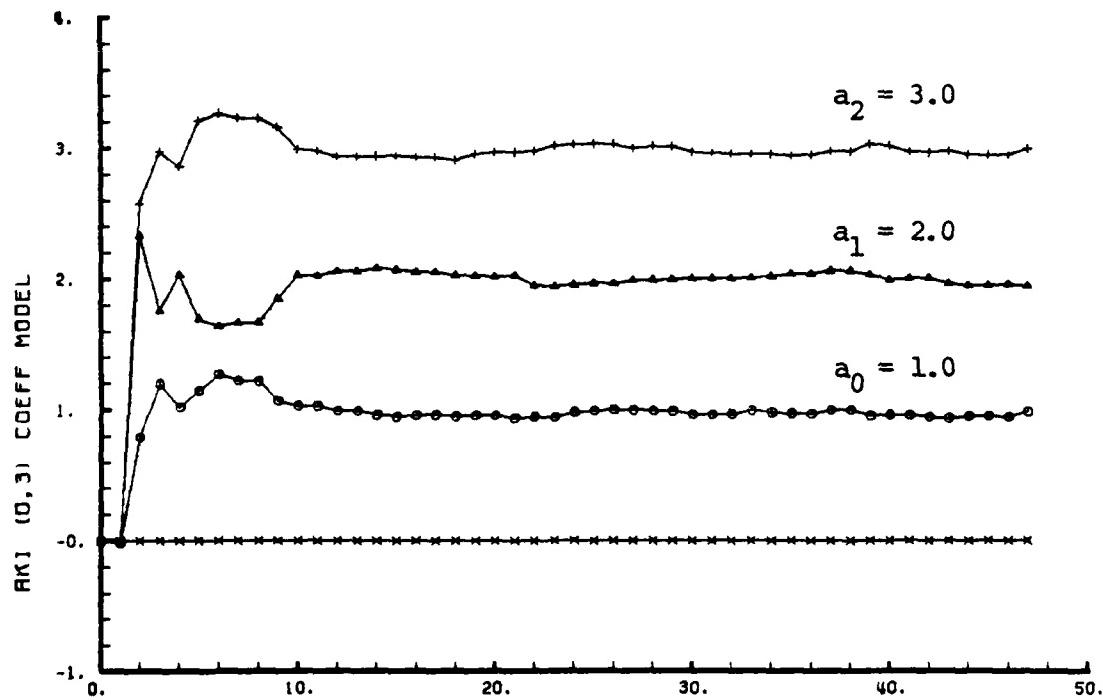
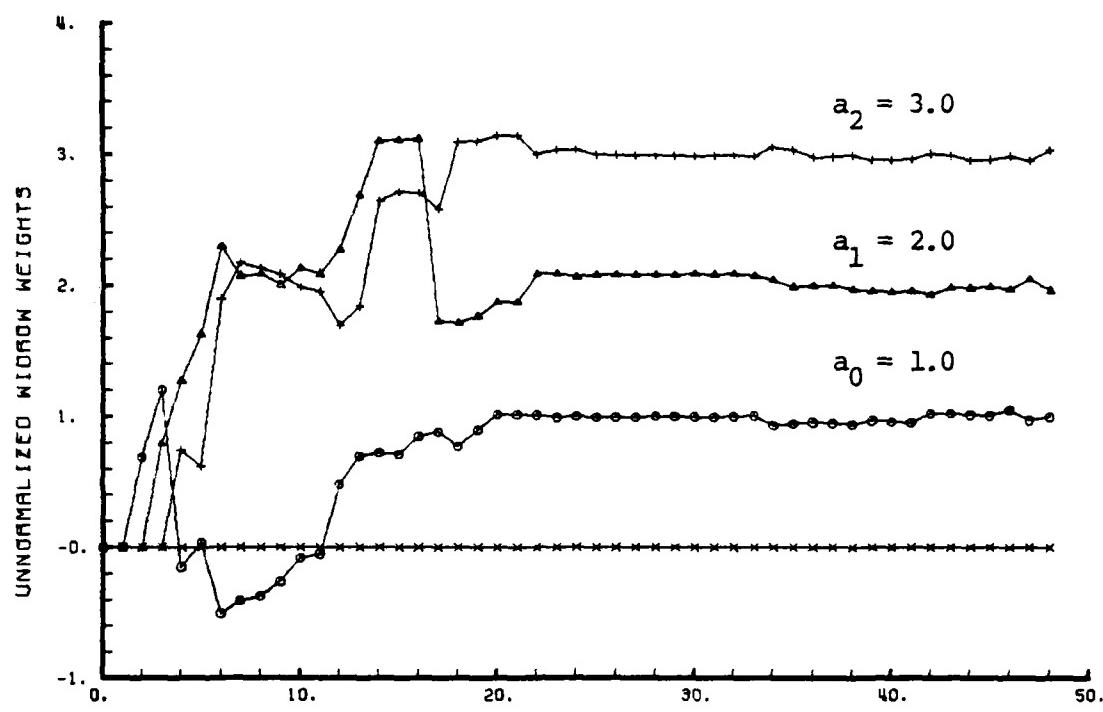


Figure 8.5b. LMS Adaptive Filter Weights,  $k_s = -.200$ , with increased measurement noise

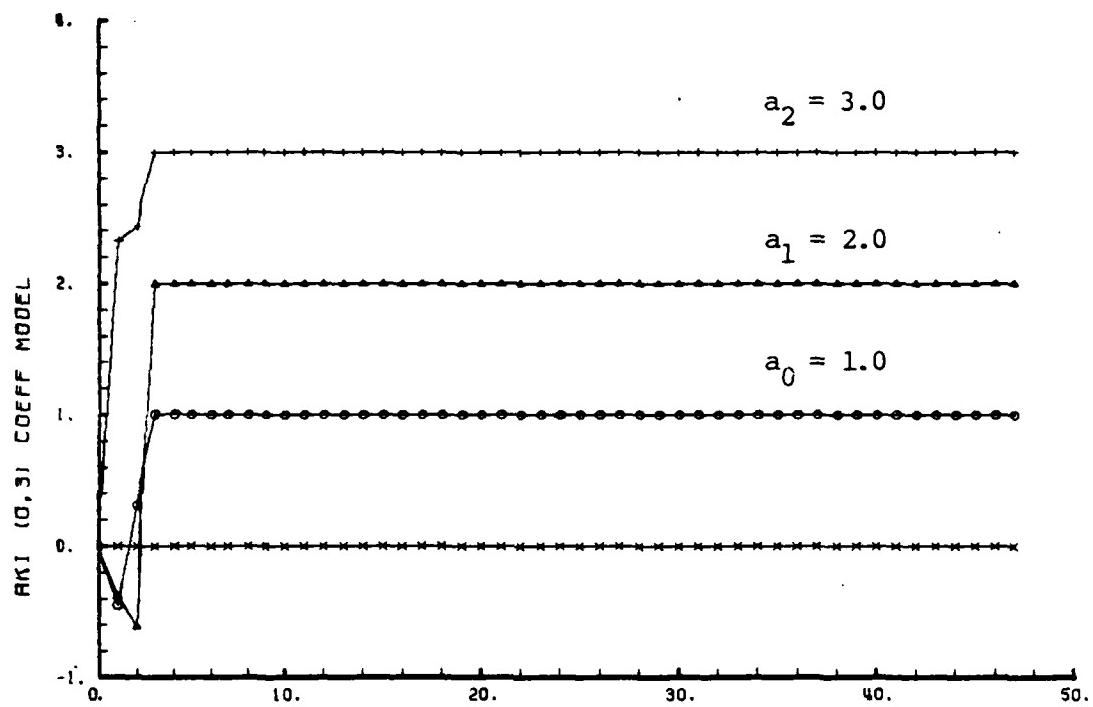


Figure 8.6. AKI Moving Average Plant Coefficients

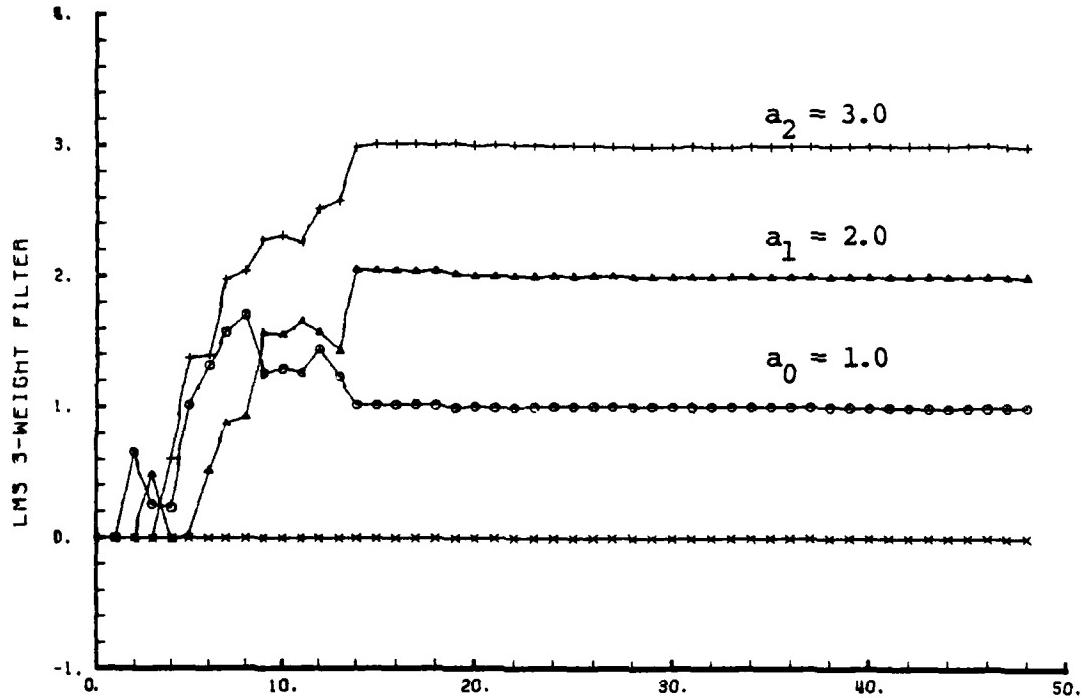


Figure 8.7. LMS Adaptive Filter Weights

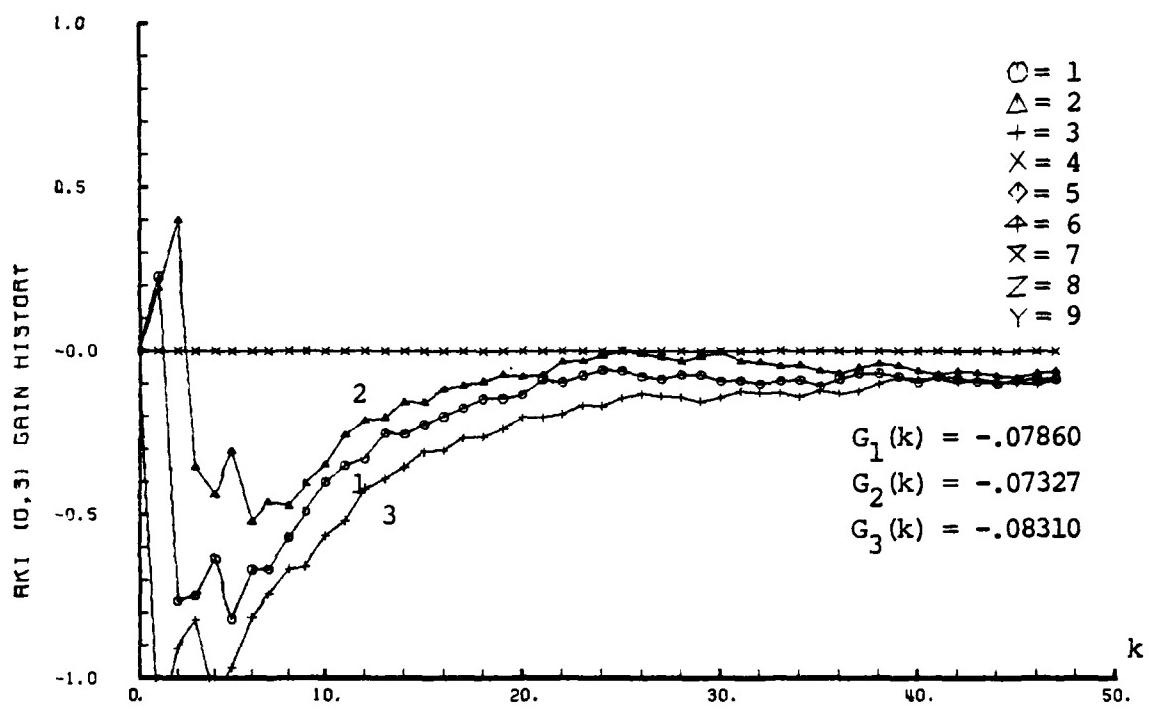


Figure 8.8. AKI Averaged Gains for Moving Average Model

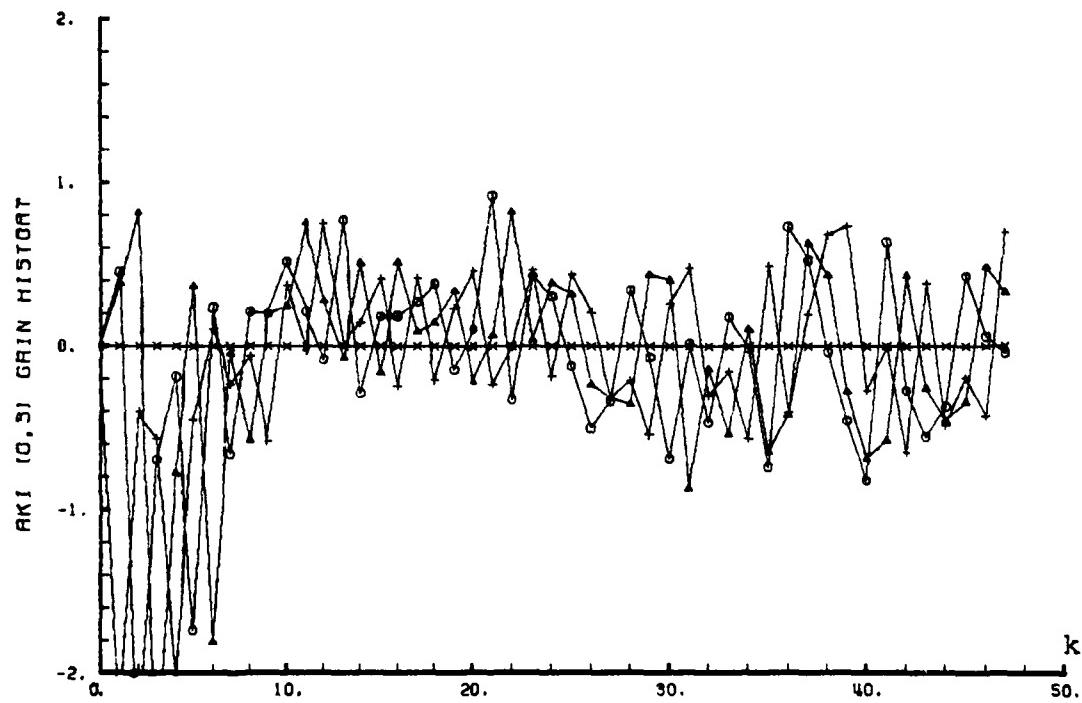


Figure 8.9. AKI Moving Average Instantaneous Gains

$E\{\underline{G}(k)\}$  is the present average value for the gain

$\underline{G}(k)$  is the present value of the instantaneous gains

$\frac{1}{k+1}$  is the averaging gain.

This method of obtaining a "running" average is generally well known, (see for example Ref. 39). The example presented was not the only one used but a good representative of the operation of the AKI when the data was derived from a moving average process.

## 2. AKI Applied to Autoregressive Data

Before applying the AKI to identifying the coefficients of a complex ARMA plant, it was first tested using data derived from a plant whose transfer function specified an autoregressive process. The transfer function used was,

$$\begin{aligned} H(z) &= \frac{1}{1 + \frac{26.0}{24.0}z^{-1} + \frac{9.0}{24.0}z^{-2} + \frac{1.0}{24.0}z^{-3}} \\ &= \frac{a_0}{1 - b_1 z^{-1} - b_2 z^{-2} - b_3 z^{-3}} \end{aligned} \quad (8.3)$$

The AKI had no problem converging on the plant coefficients (Figure 8.10) producing the following estimates at the 42<sup>nd</sup> iteration (Table 8.1). It essentially converged on the plant coefficients in approximately five iterations. The gains computed by the AKI for this case also displayed convergence to a steady state value. The gain history for this example is depicted in Figure 8.11.

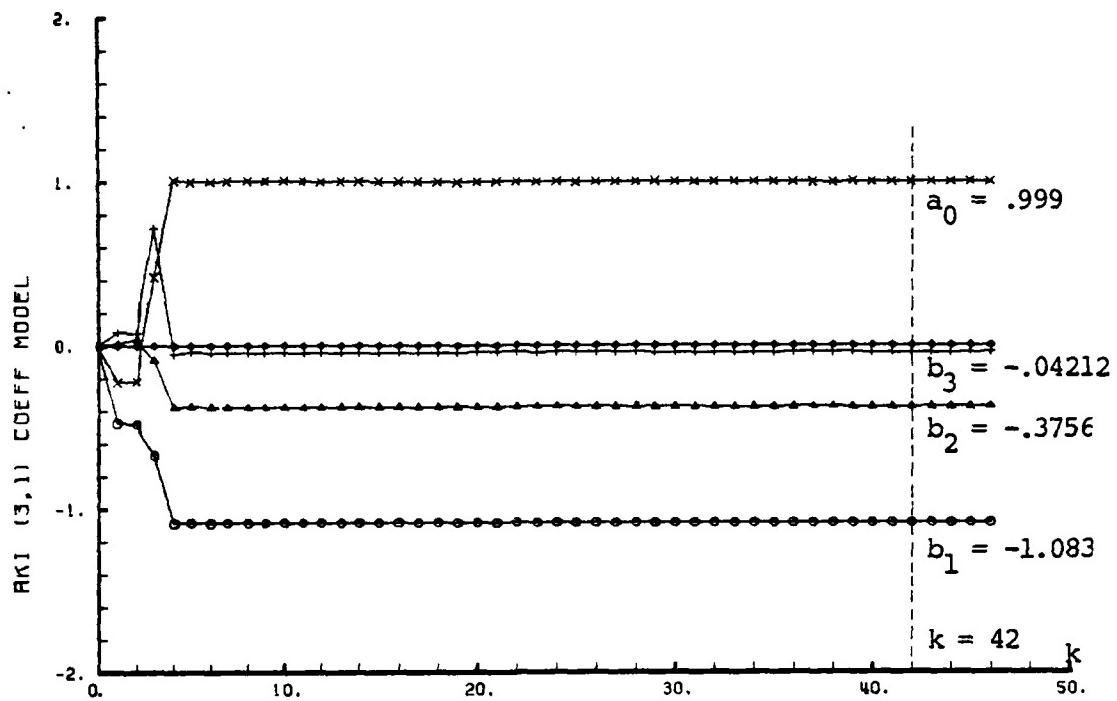


Figure 8.10. AKI Autoregressive Plant Coefficients

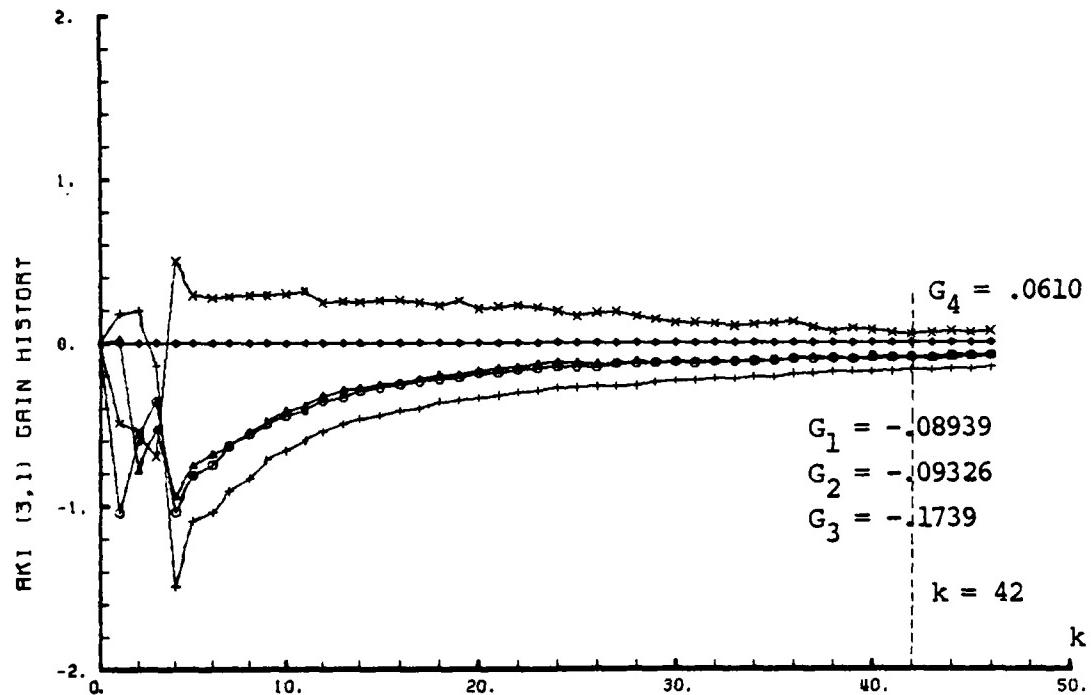


Figure 8.11. AKI Autoregressive Averaged Gains

Table 8.1  
ACTUAL VS AKI ESTIMATES FOR COEFFICIENTS OF EQUATION 8.3

	ACTUAL	AKI ESTIMATES
$a_0$	1.0	0.9995
$b_1$	-1.08333 ...	-1.083
$b_2$	-0.3750	-0.3756
$b_3$	-0.04166 ...	-0.04212

### 3. AKI Applied to ARMA Data

The next logical step was to use the AKI to identify the coefficients of a general ARMA process. One of Perry's models [Ref. 6] was used for this purpose. Specifically, the transfer function of the plant was,

$$\begin{aligned} H(z) &= \frac{1.0 + 1.4z^{-1} + .98z^2}{1 - 1.14z^{-1} + 1.4549z^{-2} - .88490z^{-3} + .40745z^{-4}} \\ &= \frac{a_0 + a_1 z^{-1} + a_2 z^{-2}}{1 - b_1 z^{-1} - b_2 z^{-2} - b_3 z^{-3} - b_4 z^{-4}} \end{aligned} \quad (8.4)$$

The ARMA plant was subjected to the same conditions which Perry [Ref. 6] describes. That is, unit variance, zero mean, white gaussian noise was used as the input. The reader is reminded that the output signal processed by the AKI was corrupted by measurement noise as described in Chapter V. The output data used in Perry's examples, however, reflects noiseless measurements. Table 8.2 tabulates the results for the coefficient estimates computed by the AKI. Even though the results presented in Table 8.2 represent the coefficient

Table 8.2

ACTUAL VS AKI ESTIMATES FOR COEFFICIENTS OF EQUATION  
8.4 AT THE 371ST ITERATION

	ACTUAL	AKI ESTIMATES
$a_0$	1.00000	0.98760
$a_1$	1.40000	1.40100
$a_2$	0.98000	0.99280
$a_3$	0.00000	0.01252
$a_4$	0.00000	-0.00922
$b_1$	1.14000	1.14000
$b_2$	-1.45490	-1.45900
$b_3$	0.88490	.88740
$b_4$	-0.40745	-.40980

estimates at the 371<sup>st</sup> iteration, it can be seen from Figure 8.12 that the AKI has essentially converged by the 28<sup>th</sup> iteration. We note also the characteristic convergence of the averaged gains to steady state values in Figure 8.13. As a means of comparison with Perry's results, the poles and zeros of the AKI estimates at the 28<sup>th</sup>, 90<sup>th</sup> and 371<sup>st</sup> iteration are plotted in Figure 8.14a and Figure 8.14b. Perry's results Figure 3.7 [Ref. 6:pp. 107,108] for his lattice modeling of the plant represented by equation (8.4) are reproduced for convenience.

As was noted for the previous cases, the instantaneous gains appeared erratic, Figure 8.15, whereas the averaged gains converged to some steady state value.

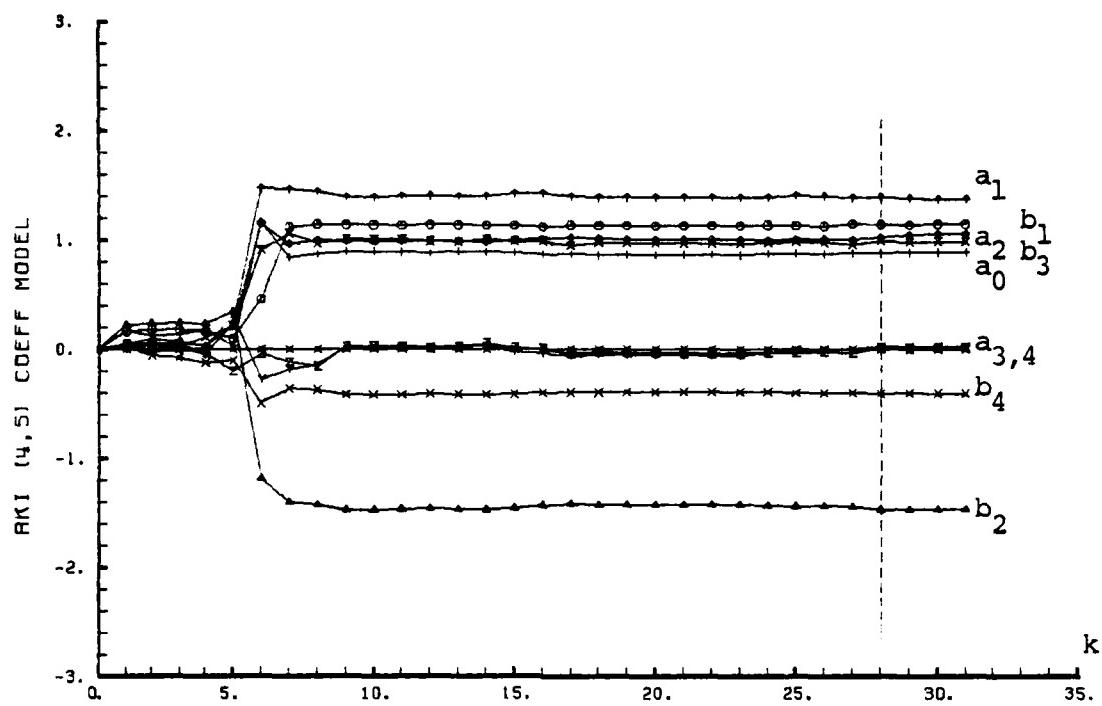


Figure 8.12. AKI(4,5) Computer Coefficients, Perry's Example

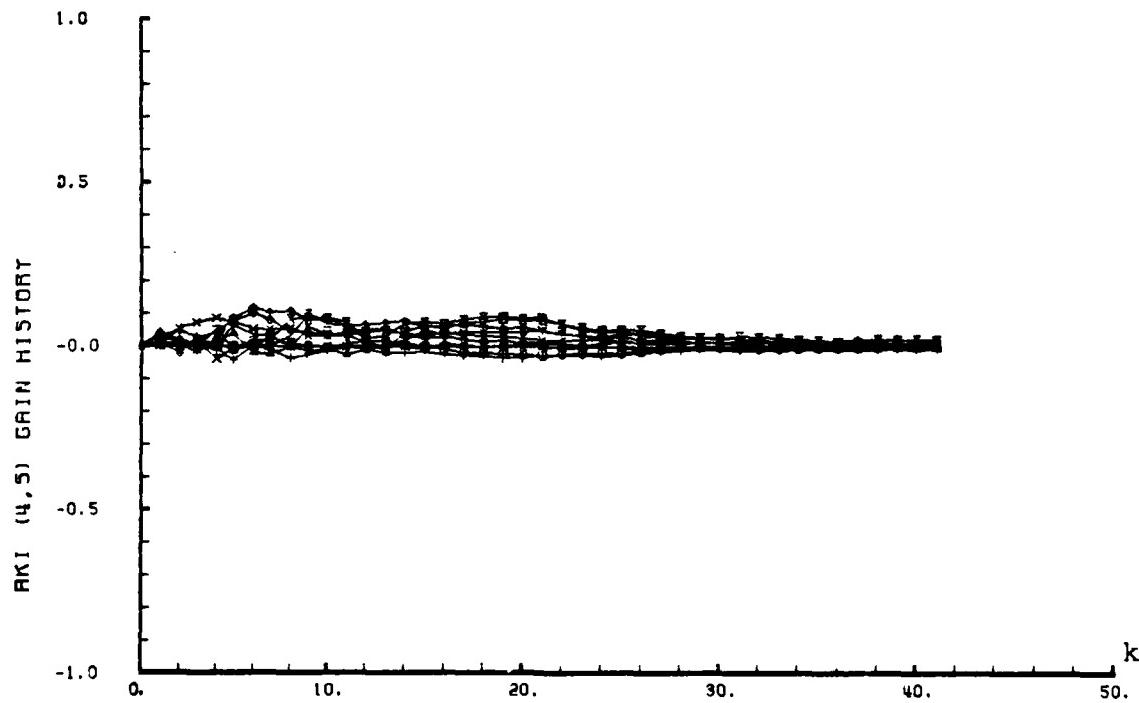


Figure 8.13. AKI(4,5) Averaged Gain Values

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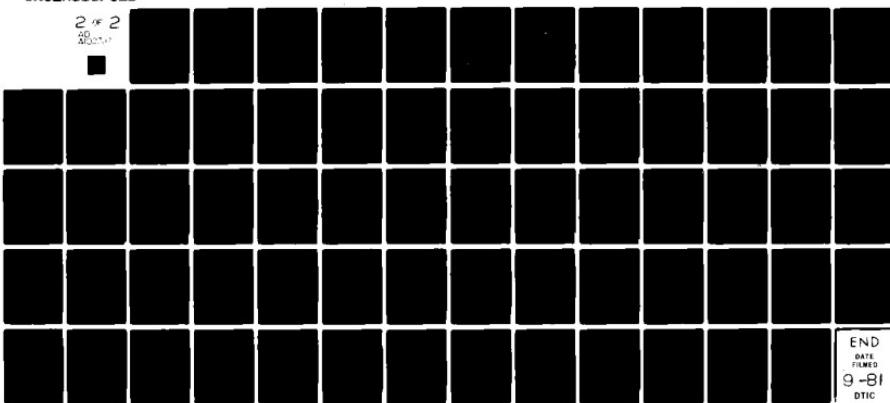
NAVAL POSTGRADUATE SCHOOL MONTEREY CA  
AN ADAPTIVE KALMAN IDENTIFIER AND ITS APPLICATION TO LINEAR AND ETC(U)  
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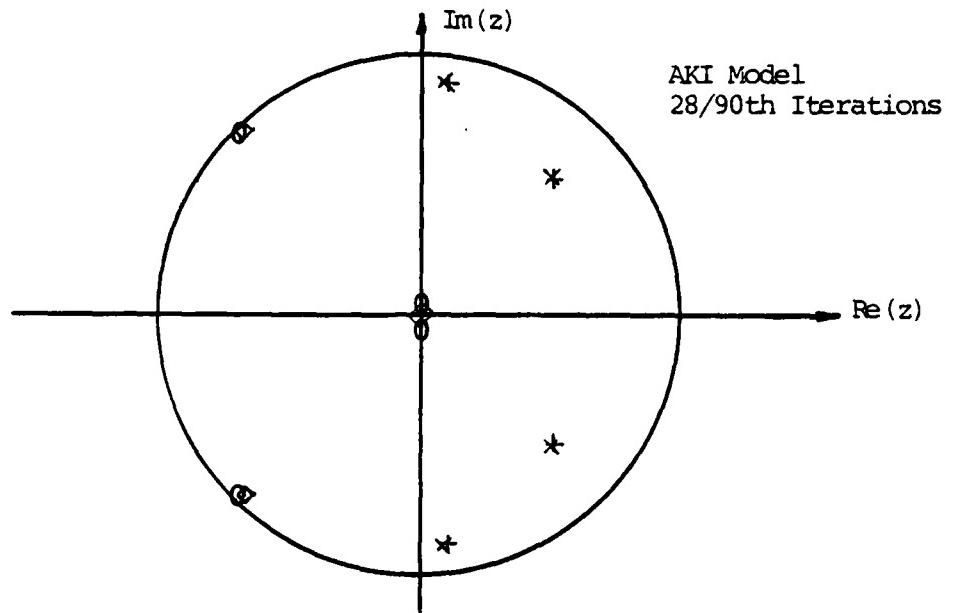
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X Model Pole  
0 Model Zero  
+ System Pole  
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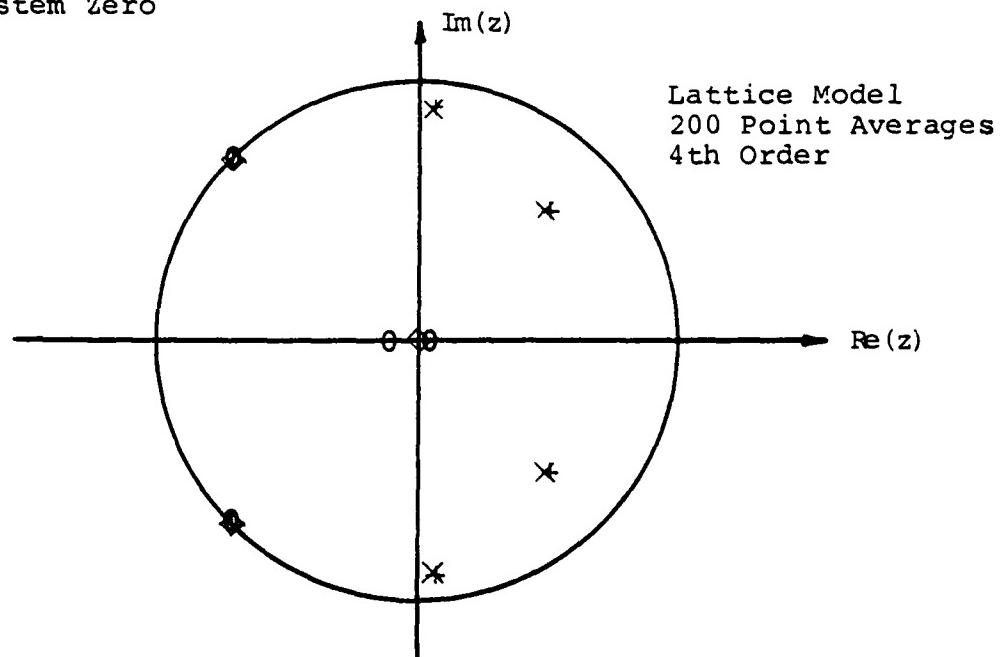
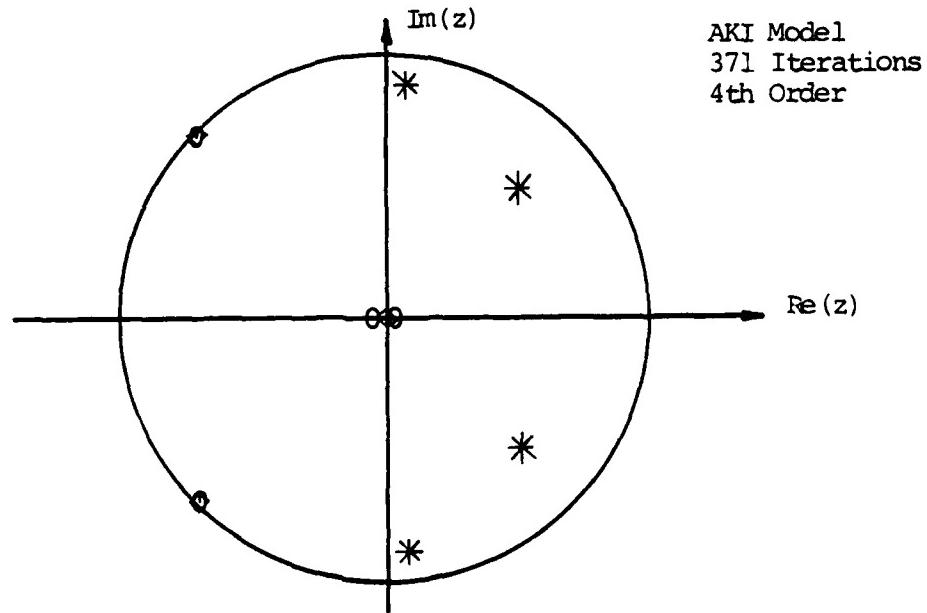


Figure 8.14a. Pole/Zero Models Produced by AKI and Lattice Algorithms for a Plant with the Characteristic Transfer Function of Eqn. (8.4)



X Model Pole  
0 Model Zero  
+ System Pole  
◊ System Zero

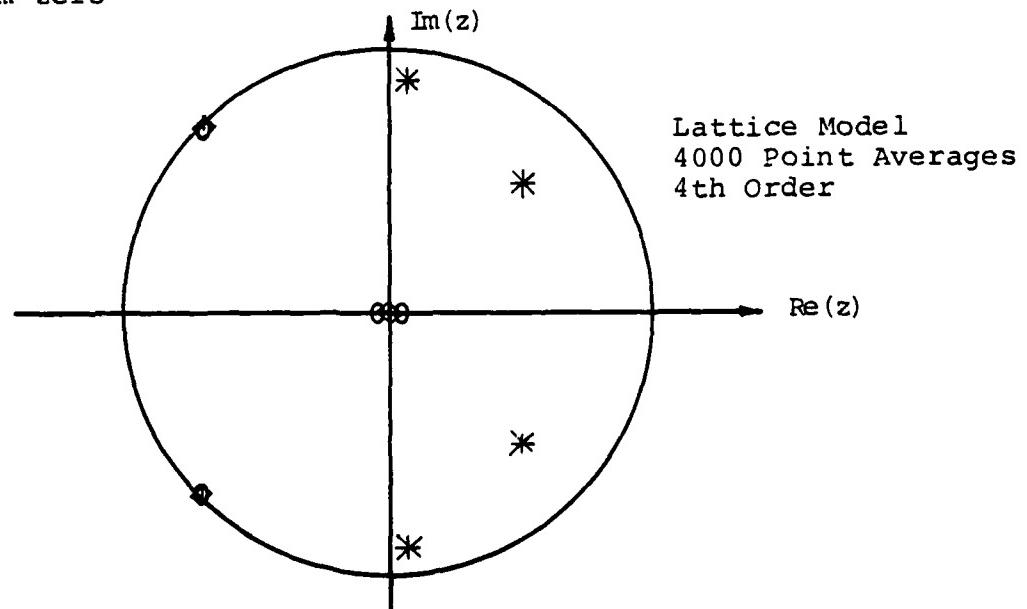


Figure 8.14b. Pole/Zero Models Produced by AKI and Lattice Algorithms for a Plant with the Characteristic Transfer Function of Eqn. (8.4) at 371st Iteration

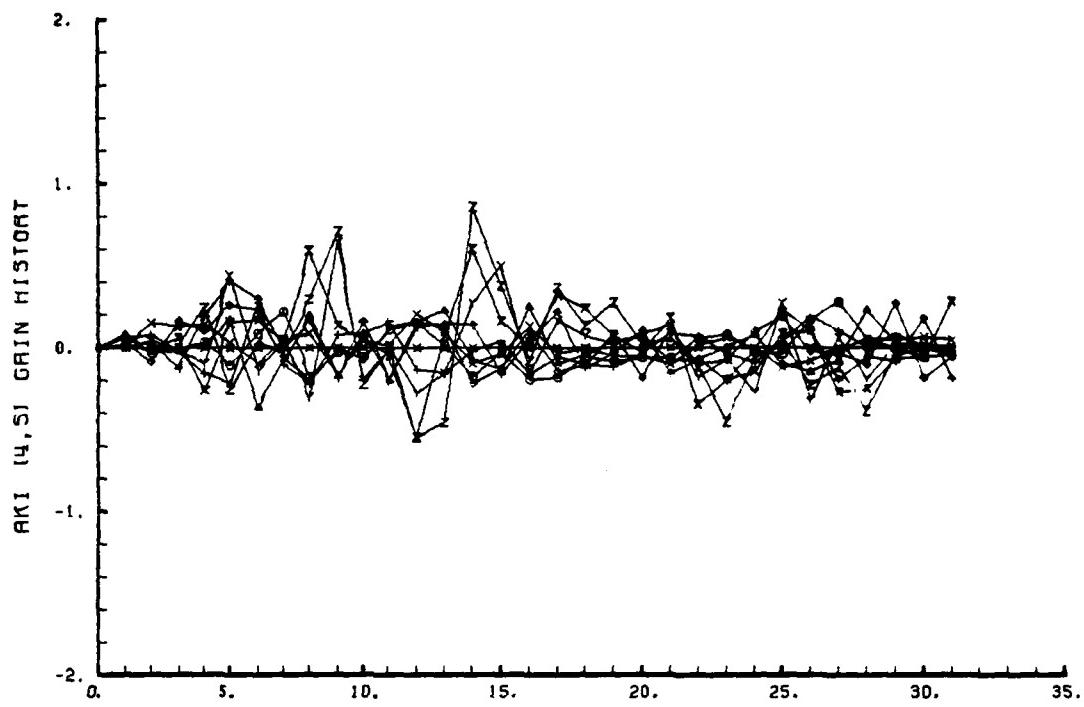


Figure 8.15. Instantaneous Gain Values, AKI(4,5) Model

#### 4. AKI vs Adaptive Recursive LMS Filter

Using Feintuch's proposed algorithm [Ref. 18] and repeating the simulation presented in the rebuttal to Johnson and Larimore [Ref. 19], a comparison was made between the operation of the AKI and the Adaptive Recursive LMS filter. The Adaptive Recursive LMS filter required "tuning" of the convergence factors,  $k_a$  and  $k_b$ , to what appeared to be the optimal performance features of (1) fast convergence and

(2) stable estimates. The "tuning" process was a trial and error procedure beginning with the convergence constants given by Feintuch,

$$k_a = -4.3 \times 10^{-4} \quad (8.5a)$$

$$k_b = -4.3 \times 10^{-4}. \quad (8.5b)$$

The Adaptive Recursive LMS algorithm was then implemented to estimate the coefficients of the plant whose transfer function was,

$$H(z) = \frac{0.05 - 0.40z^{-1}}{1 - 1.1314z^{-1} + 0.25z^{-2}} \quad (8.6a)$$

$$= \frac{a_0 + a_1 z^{-1}}{1 - b_1 z^{-1} - b_2 z^{-2}} \quad (8.6b)$$

The best response obtained by trial and error resulted in the convergence constants,

$$k_a = -4.3 \times 10^{-3} \quad (8.7a)$$

$$k_b = -14.3 \times 10^{-3}. \quad (8.7b)$$

Feintuch reported that after  $8,192^2$  iterations the estimates had converged on the coefficients of (8.6a) with a .2096 normalized rms error. Using the convergence constants (8.7) convergence was essentially reached by the 4,000<sup>th</sup> iteration.

<sup>2</sup>Feintuch reports the resulting estimates for the coefficients at several intermediate iterations from 8,192 to 65,536, however 8,192 was the minimum number of iterations reported.

It was felt that the operation of the adequately tuned Adaptive Recursive LMS filter could be fairly compared with the operation of the AKI.

Table 8.3 tabulates the performance of the two system identification methods and Figures 8.16 and 8.17 graphically present the responses. Figure 8.18 not only shows at what point the gains of the AKI reach a constant value but also

Table 8.3

ACTUAL VS AKI AND ADAPTIVE RECURSIVE LMS FILTER ESTIMATES FOR COEFFICIENTS OF EQUATION 8.6

	ACTUAL	AKI ESTIMATES (k = 150)	ADAPTIVE RECURSIVE LMS (k = 3,990)
a <sub>0</sub>	0.05	0.05107	0.05045
a <sub>1</sub>	-0.40	-0.4007	-0.3990
b <sub>1</sub>	1.1314	1.13200	1.12800
b <sub>2</sub>	-0.25	-0.25090	-0.2442

provides a measure of confidence that the unknown system has been identified. It is evident from Figure 8.19 that both methods seem to identify the poles and zeros of the actual plant; however, the poles computed by the AKI are closer. The zeros computed by the AKI are approximately .080 units farther from the true zero than is the Adaptive Recursive LMS filter estimate. All aspects considered, the AKI takes considerably fewer iterations to arrive at its estimate.

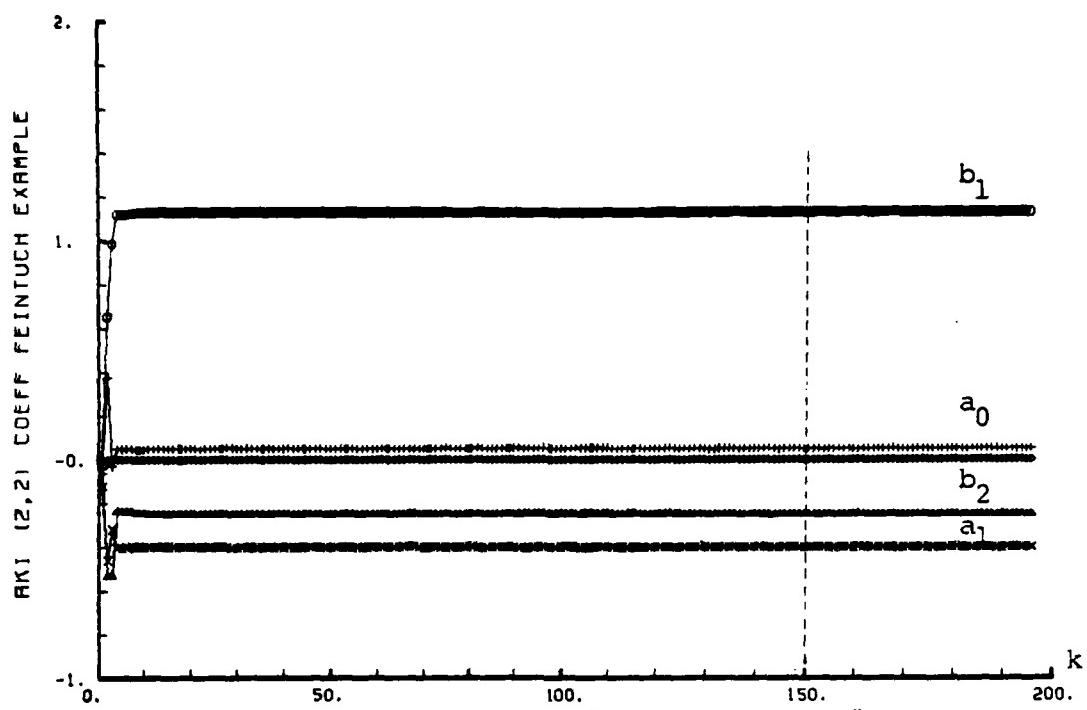


Figure 8.16. AKI(2,2) Computed Coefficients, Feintuch Example

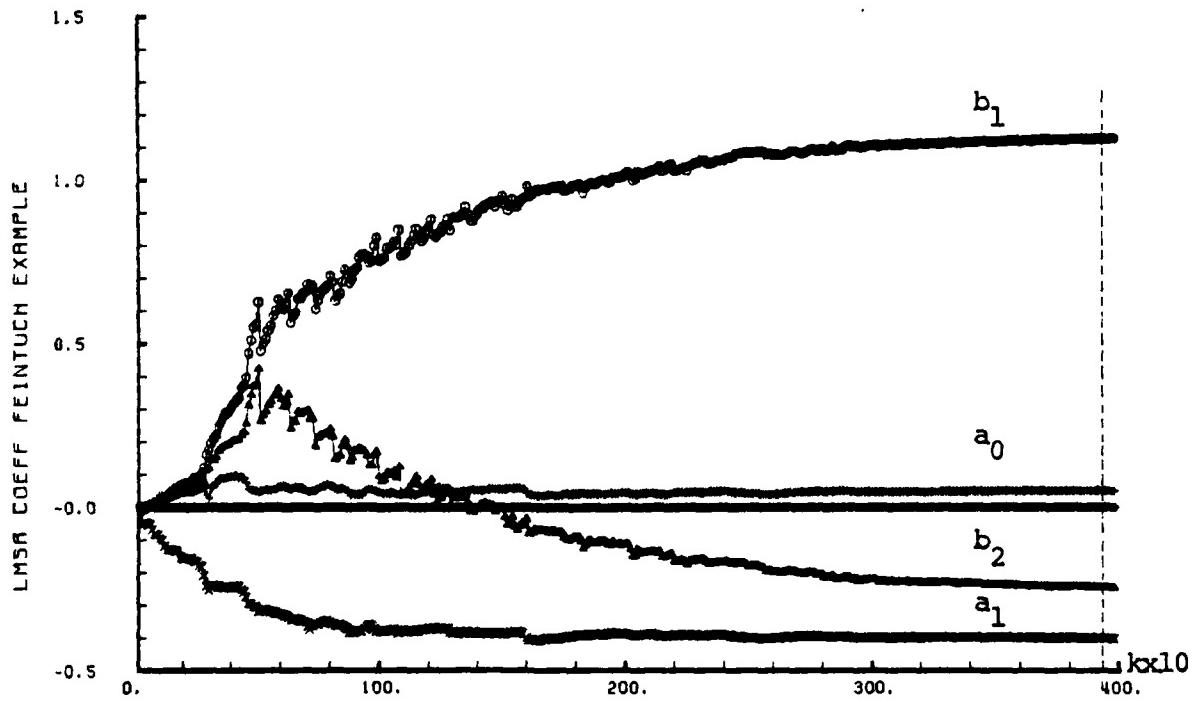


Figure 8.17. LMSR Computed Coefficients, Feintuch Example

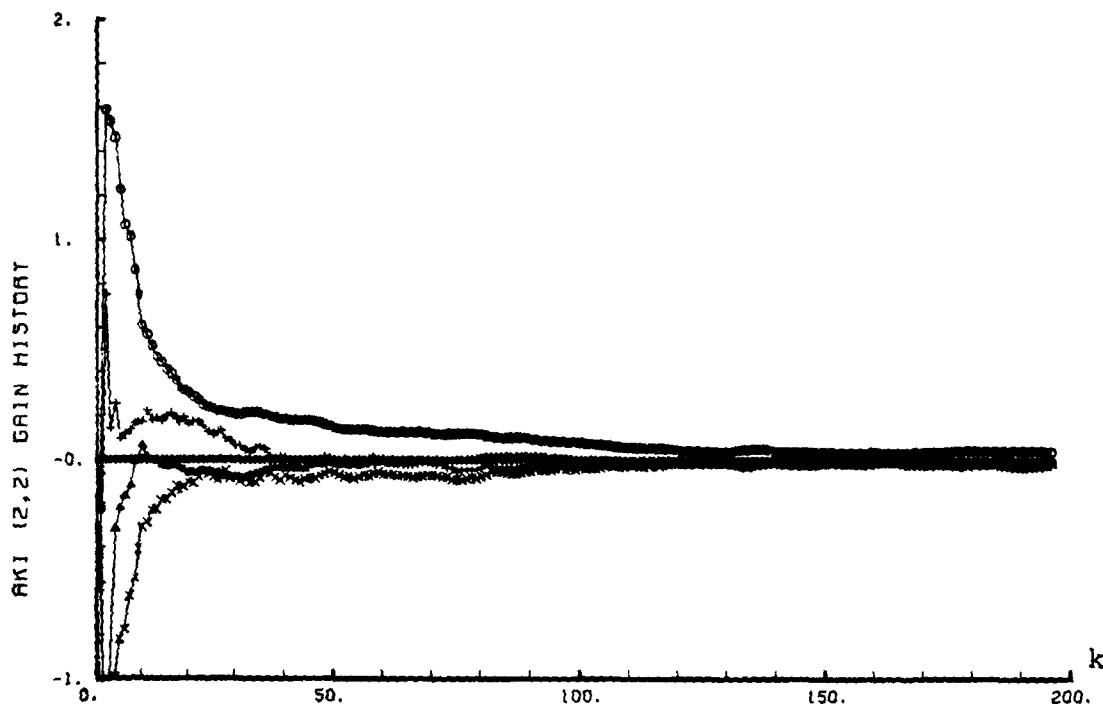


Figure 8.18. AKI(2,2) Averaged Gains, Feintuch Example

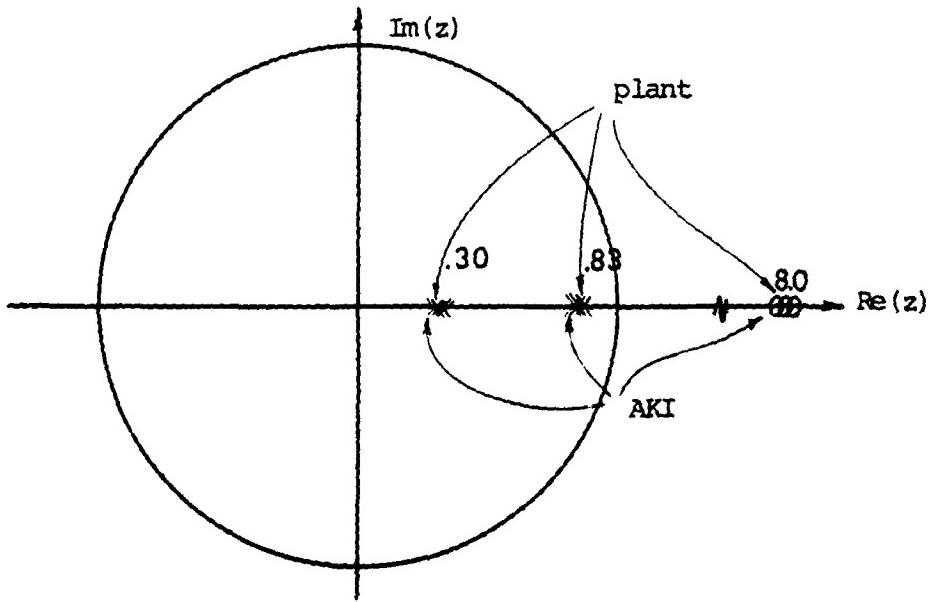


Figure 8.19. Pole/Zero Models Produced by AKI and Adapt.  
Rec. LMS Algorithms for a Plant with the  
Characteristic Transfer Function of Eqn. (8.6)

### 5. AKI Applied to PLL Data (Linear Region)

To identify the coefficients associated with the general autoregressive-moving average representation for the phase locked loop, the input noise signal power (mean square value) was kept at  $(10 \text{ deg})^2$ . In this manner, the linear region of the PLL was invoked. The input/output data was analyzed by the AKI resulting in the estimates shown in Table 8.4. Figures 8.20 and 8.21 show the response of the AKI

Table 8.4

#### ACTUAL VS AKI ESTIMATES OF THE ARMA REPRESENTATION FOR THE PLL (LINEAR REGION)

	ACTUAL	AKI ESTIMATES ( $k = 82$ )	AKI ESTIMATES ( $k = 350$ )
$a_0$	0.000000	-0.0001046	-0.000692
$a_1$	0.020000	0.0198500	0.0199800
$a_2$	-0.018890	-0.018720	-0.0188900
$b_1$	1.980000	1.980000	1.978000
$b_2$	-0.981110	-0.981000	-0.9790

when applied to the identification of the PLL data. It was noted that though the AKI correctly identified the coefficients of the linear PLL, the AKI gains were large due to the weak signals (input/output data) incorporated in the measurement vector,  $H(k)$ .

### 6. AKI Applied to PLL Data (Non-linear Region)

When analyzing any non-linear system the engineer must bring to bear all his analysis techniques on the problem. The PLL was therefore studied using classical root locus

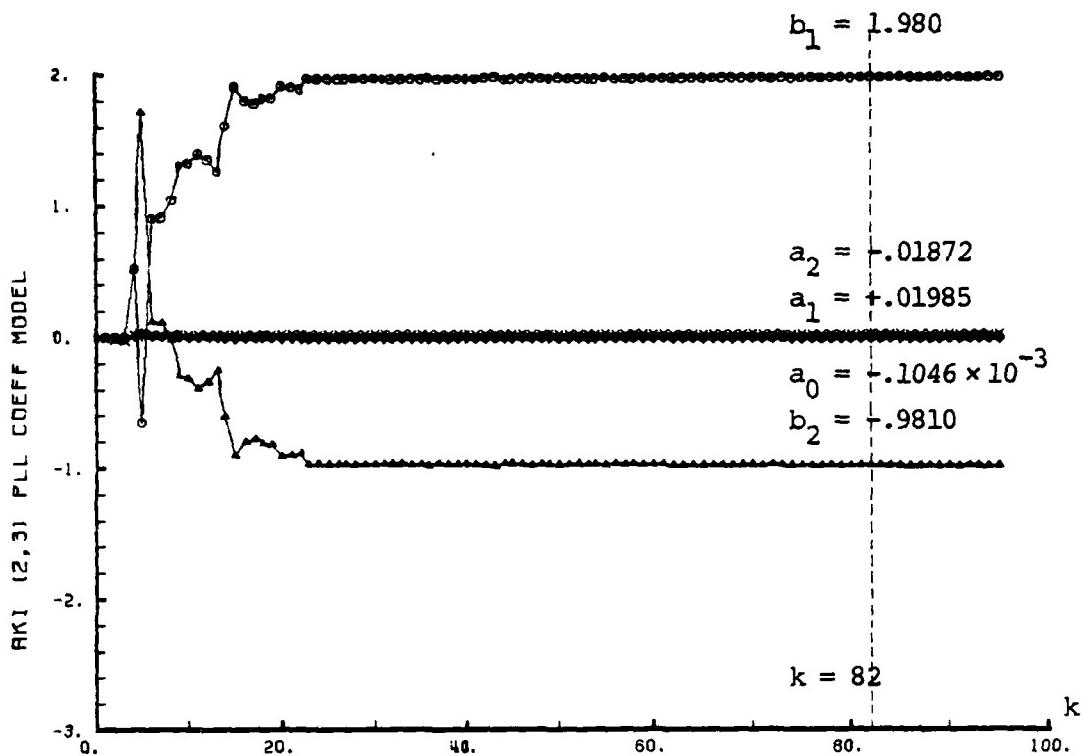


Figure 8.20. PLL, AKI(2,3) Computed Coefficients

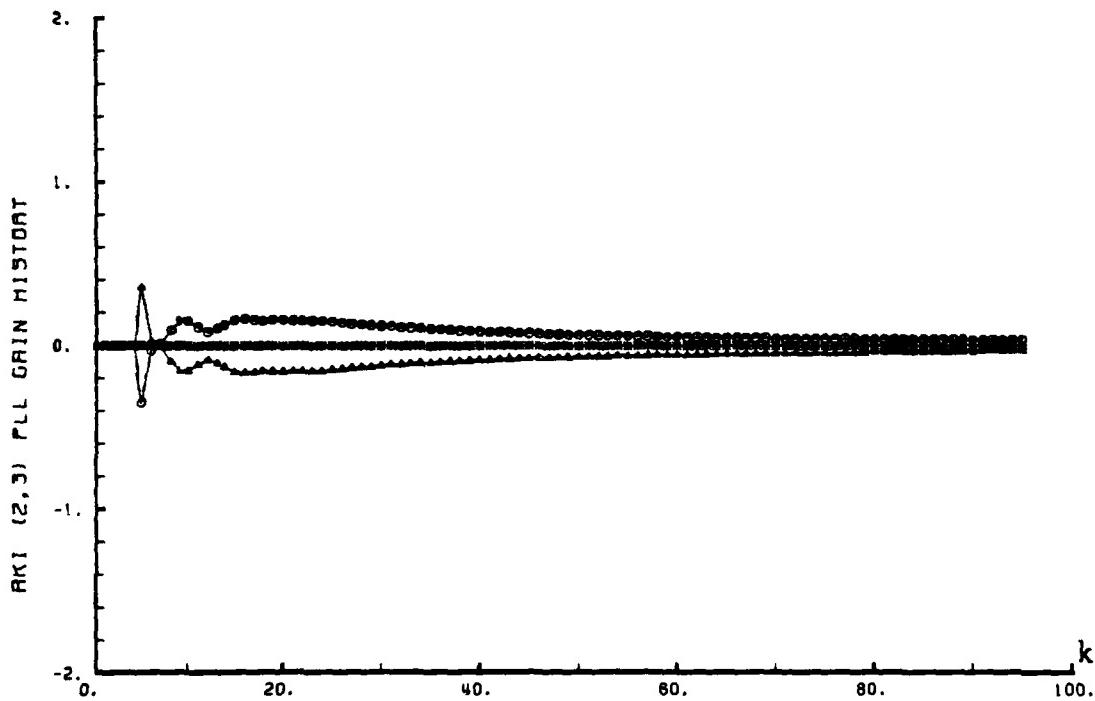


Figure 8.21. Averaged Gain Values, AKI(2,3) PLL Model

techniques to preview the possible outcomes when being modeled by the AKI. Stability analysis of a more complex PLL system using the root locus technique has been previously presented in the literature [Ref. 40].

The root locus technique can be applied to the PLL presented in this thesis by first assuming that the  $\sin(\cdot)$  block of Figure 5.7 is a variable gain,  $\lambda$ . This is not a restrictive assumption since the PLL during operation generally tracks small deviations. The loop gain can be written by inspection as,

$$\begin{aligned} L(z) &= \frac{\lambda(0.02)(1 - 0.9445z^{-1})z^{-1}}{(1 - z^{-1})^2} \\ &= \frac{\lambda(0.02)(z - 0.9445)z}{(z - 1)^2} \end{aligned} \quad (8.8)$$

From equation (8.8) the root locus of  $L(z)$  is drawn as shown in Figure 8.22. Even though the root locus technique is generally used when the signals in the system are considered deterministic, it is not surprising that some of the results obtained are nevertheless valid. When a moderately strong input noise identification signal [ $E\{u^2(u)\} < (25 \text{ deg})^2$ ] was used, the pole-zero locations of the PLL seem to follow the classical root locus analysis. However, when the input noise identification signal power is increased beyond  $(25 \text{ deg})^2$  the pole-zero locations do not follow the expected behavior predicted using the root locus method as can be seen in Figure 8.22.

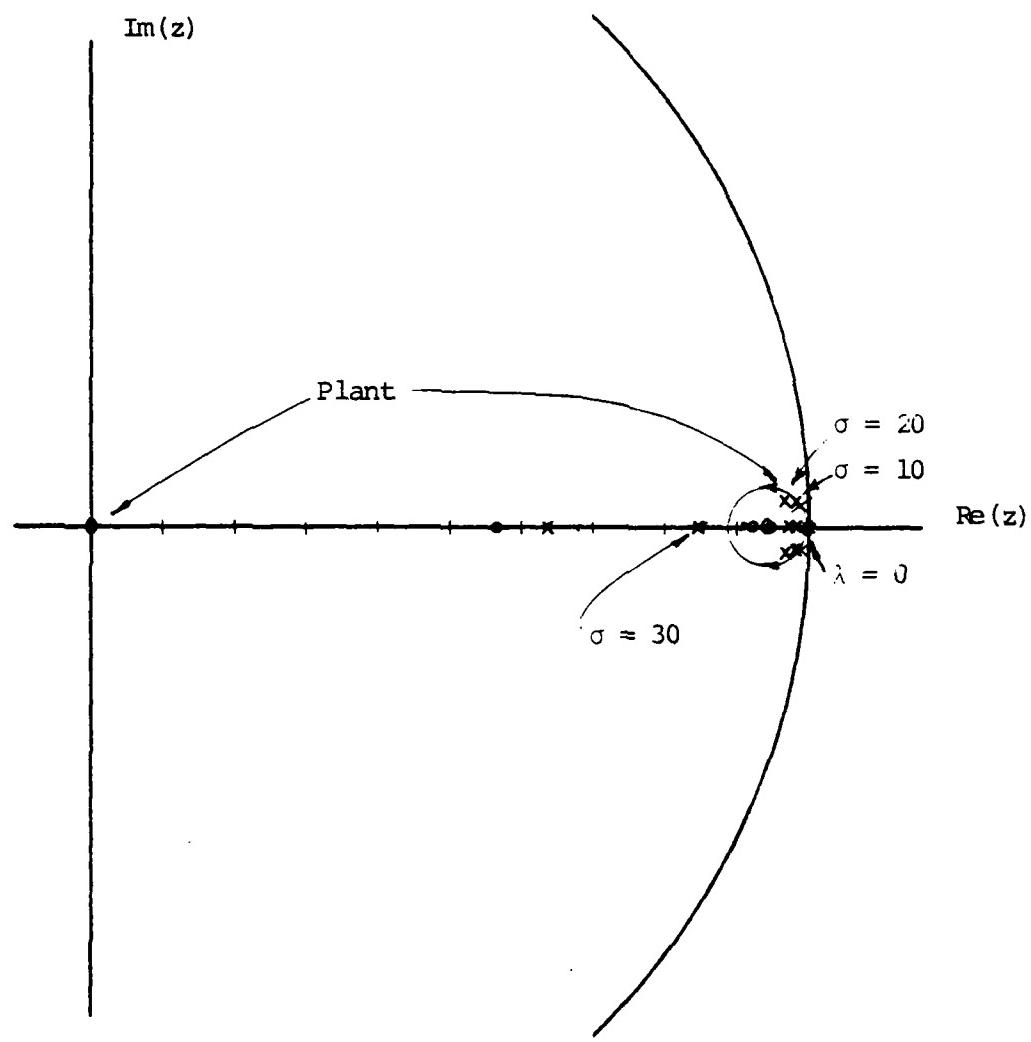


Figure 8.22. PLL Root Locus Analysis vs Computer Roots  
of the AKI ARMA Model

The departure of the pole-zero behavior from what was expected was analyzed by closer investigation of the linear terms of equation (7.8).

$$\begin{aligned} y(k) &= .02\alpha u(k-1) - .01889\alpha u(k-2) + (2.0 - .02\alpha)y(k-1) \\ \text{lin} &+ (.01889\alpha - 1)y(k-2) \end{aligned} \quad (8.9)$$

Recalling that a third order Taylor series approximation of the sine is the functional expressed by equation (7.7),

$$\sin(x) \approx \alpha x + \beta x^3 \quad (7.7)$$

one notes that the linear region is described when  $\alpha = 1$  and  $\beta = 0$ . It is therefore reasonable to study the variation of  $\alpha$  with respect to input noise power.

The average value  $\bar{\alpha}$  of  $\alpha$ , was computed by equating the estimated AKI coefficients to the coefficients of like terms in equation (8.9) for several input noise power levels. The relation used was

$$\bar{\alpha} = \frac{1}{4} \left[ \frac{2.0 - \hat{b}_1}{.02} + \frac{1.0 + \hat{b}_2}{.01889} + \frac{\hat{a}_1}{.02} - \frac{\hat{a}_2}{.01889} \right] \quad (8.10)$$

at the 350th iteration. The relationship between  $\bar{\alpha}$  and the input noise power level is readily apparent from Figure 8.23. This result is plausible since if one considers the input-output relationship of the sine block of Figure 5.7 one obtains Figure 8.24. Superimposing the gaussian probability functions of the different input noise signals, it can be seen that for

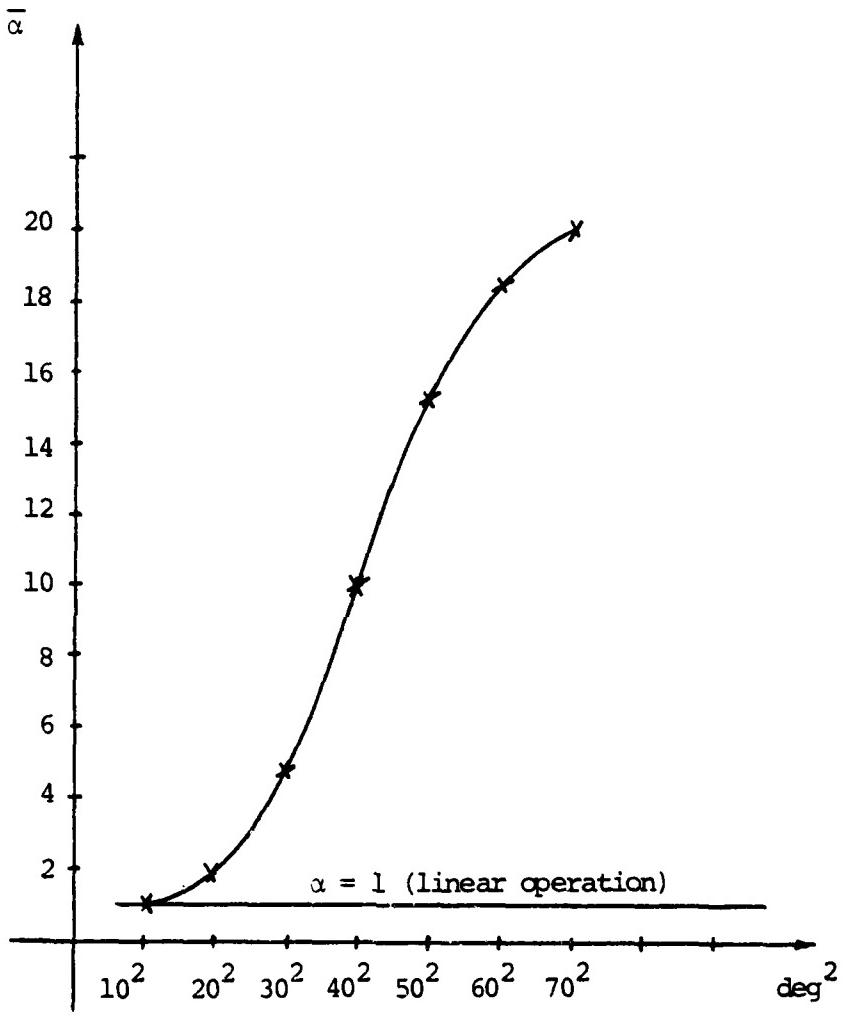


Figure 8.23. Variation of  $\bar{\alpha}$  vs Input Noise Signal Power

low power levels of the input signal, identification of the linear parameters of the overall system (equation 5.9) can be made. As the input power is increased beyond (25 deg)<sup>2</sup> the input-output characteristics of the sin x block are no longer approximately linear, showing its effect in Figure 8.24 as a departure from its linear operation,  $\bar{\alpha} = 1$ . Therefore, by monitoring  $\bar{\alpha}$  one can determine when the overall system is entering its non-linear operating regime.

Since the same functional dependence between each of the AKI estimates and  $\bar{\alpha}$  did not exist for all of the input noise powers considered, knowing  $\bar{\alpha}$  did not provide any information of the pole locations but did provide a measure of the degree of non-linear operation.

#### B. ORDER OF THE UNKNOWN SYSTEM IS NOT KNOWN (OVERMODELING)

##### 1. AKI vs LMS Adaptive Filter

Using the data derived by operating a plant with the transfer characteristic of equation (8.1) the operation of the AKI and the LMS adaptive filter was compared when the orders used in the identifier and the LMS filter were greater than the known process that generated it. The following overmodeling cases were studied:

- (1) MA model is greater than plant MA process
- (2) ARMA model is fitted to MA plant

When the MA model order is greater than that of the plant MA process, it was found that both the AKI and the LMS filter would compute coefficients close to zero for the higher

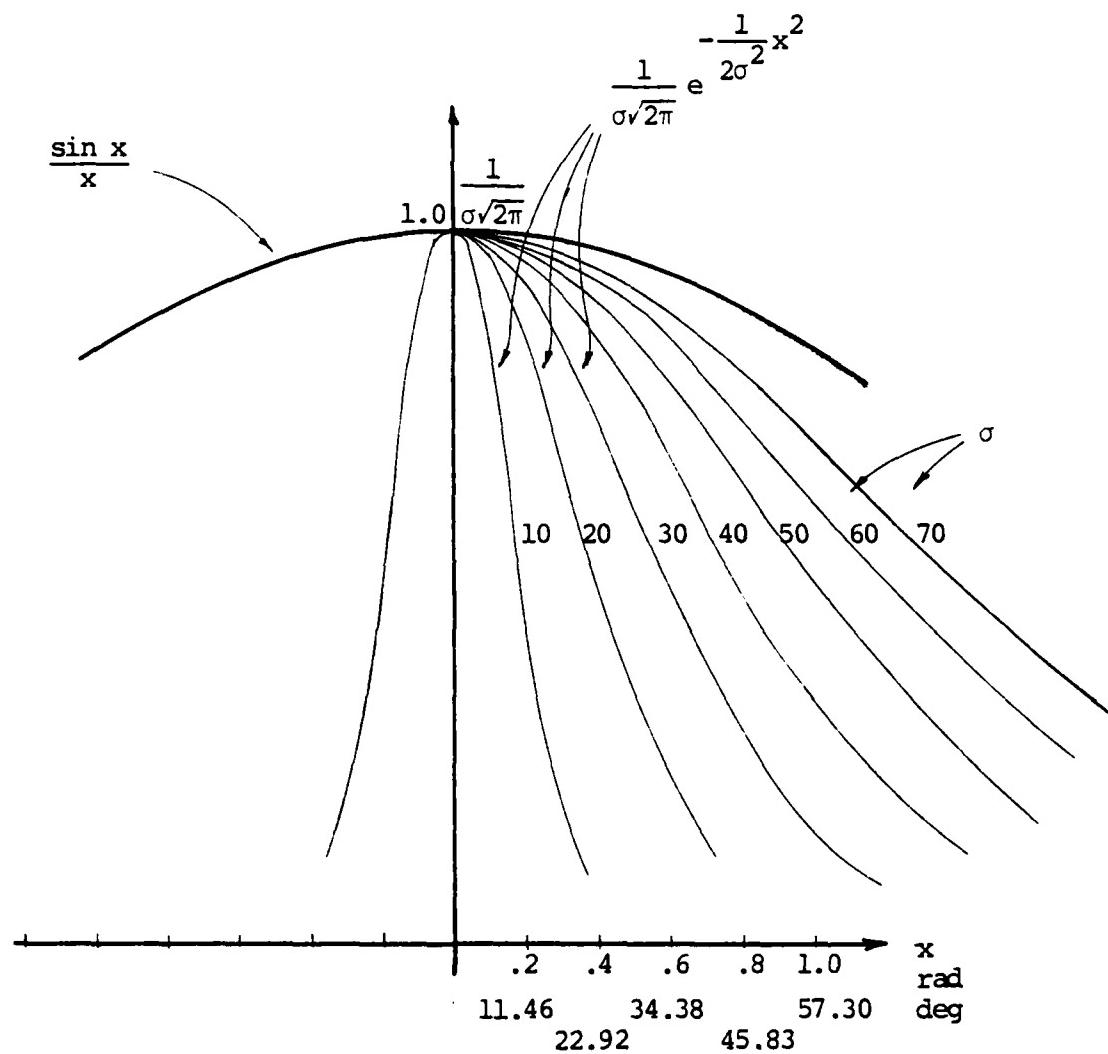


Figure 8.24. Comparison of  $\frac{\sin x}{x}$  and p.d.f. of input noise signals

order coefficients. This effect can be seen in Figures 8.25a and 8.25b when a fourth order MA process is used to model the actual second order process represented by equation (8.1). Since these are a "zeros only" plant and model the overmodeling essentially causes zeros to appear at the origin of the Z-plane of the model transfer function. Table 8.5 summarizes the results for two overmodeling conditions at the 250th iteration using the AKI. When a purely moving average process represented by equation (8.1) was modeled as an ARMA process, one must direct his attention to the poles and zeros of the model transfer function which the AKI computed and compare them to the actual plant poles and zeros. It was not readily apparent from the resulting coefficients that the plant had been identified. The following example will help clarify what is happening.

For the data produced by the second order plant (equation 8.1), an autoregressive moving average (ARMA) model of orders 2 and 5 respectively was fitted. It can be seen from Table 8.6 that no firm conclusions can be drawn about which coefficients actually identify the plant. We, the analysts, knowing the form of the plant which produced the data, could qualitatively state that  $b_1$ ,  $b_2$ ,  $a_3$ ,  $a_4$  and  $a_5$  are small enough to be ignored. Hence, we can identify the plant correctly. However, inspection of the poles and zeros of the transfer function which the AKI computed,

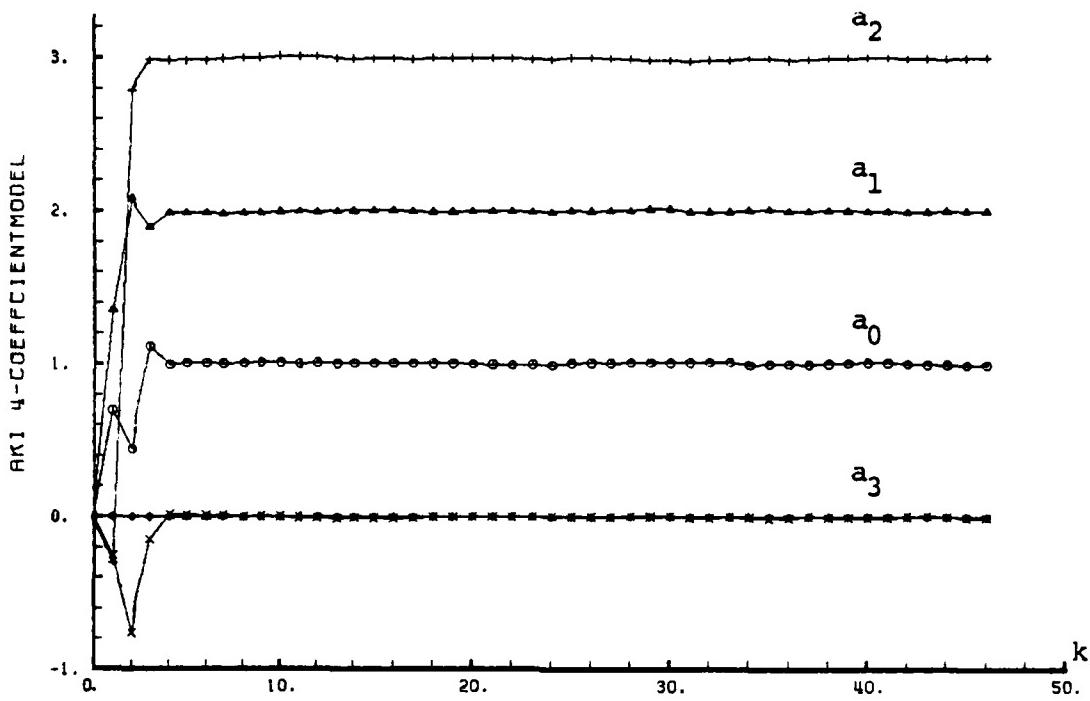


Figure 8.25a. AKI(0,4) Model of a Third Order MA Process

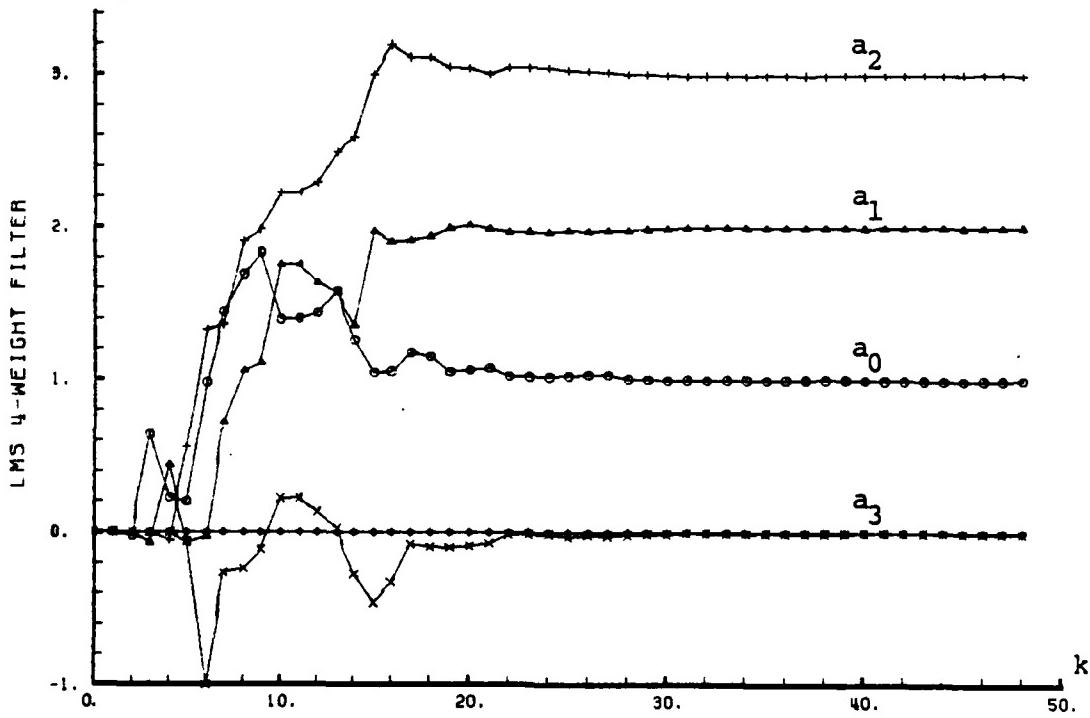


Figure 8.25b. Fourth Order LMS Filter Weights of a Third Order MA Process

Table 8.5  
OVERMODELING OF A MA PROCESS USING THE AKI ( $k = 250$ )

	Actual Model Coeffs	3rd Order AKI MA Model	5th Order AKI MA Model
$a_0$	1.0	.9995	.9991
$a_1$	2.0	2.0000	2.000
$a_2$	3.0	3.0000	3.000
$a_3$	0.0	$.893 \times 10^{-3}$	.0003191
$a_4$	0.0	---	.001082
$a_5$	0.0	---	.0003198
zeros:	$-1.0 \pm 1.414j$	$-1.000 \pm 1.414j$ $-.2977 \times 10^{-3}$	$-1.001 \pm 1.414j$ .0227 $\pm .0428j$ -.4534

Table 8.6  
OVERMODELING OF A MA PROCESS USING THE AKI OF ORDERS  
AR = 2, MA = 6

	Actual Plant Coefficients	AKI(2,6) Model Coefficients
$b_1$	0.0	.00467
$b_2$	0.0	.10500
$a_0$	1.0	1.0000
$a_1$	2.0	1.9940
$a_2$	3.0	2.8840
$a_3$	0.0	-0.2257
$a_4$	0.0	-0.3142
$a_5$	0.0	0.0001517

$$H_{AKI(2,6)}(z) = \frac{1.0 + 1.994z^{-1} + 2.8840z^{-2} - 0.2257z^{-3} - 0.3142z^{-4} + 0.001517z^{-5}}{1 - .00467z^{-1} - .1050z^{-2}}$$

(8.11)

results in:

poles:	-.8217
	.3264
zeros:	-.9996 ± 1.414j
	.4827 × 10 <sup>-3</sup>
	-.3212
	.3262

The observation to be made is that there are two pole-zero combinations which are near cancellation. This suggests that the AKI algorithm be rerun with the AR and MA orders reduced by at least two.

The implication of the analysis of this section is that a model can in theory be found for a given set of input/output data. Further, a parsimonious model can be identified by careful observation of pole-zero combinations which are near cancellation and of stray zeros near the origin.

## 2. AKI Applied to AR and ARMA Data

Essentially the same characteristic results found in Section VIII.B.1 were confirmed when the data produced by plants defined by the transfer functions of equations (8.3) and (8.4) were analyzed by the overmodeled AKI. That is, pole-zero pairs near cancellation and zeros near the origin

were produced by the AKI. Figures 8.26 through 8.28 have been chosen as representative pole-zero plots of the transfer functions computed by the AKI for the AR plant of Section VIII.A.2 and for Perry's model (Section VIII.A.3).

### C. CONCLUSION

This work indicates that the Kalman filter algorithm heretofore generally used as a state estimator, or in augmented form to estimate parameters (in which case the parameters are treated as states), can also be formulated in an adaptive manner to iteratively estimate the coefficients of an ARMA equation explicitly. This approach, termed the Adaptive Kalman Identifier (AKI), summarily identifies the unknown system whose input/output data is being processed. The LMS adaptive algorithm of Widrow, and its modification by Griffiths (in which the convergence factor is selected to be inversely proportional to the input signal power) are shown to be suboptimal cases of the AKI. An additional insight provided by the AKI is that it indicates clearly how measurement noise might be taken into account in the LMS adaptive formulation.

The operation of the AKI was checked by way of simulation and compared with two existing identification techniques: (1) the LMS Adaptive nonrecursive (MA) algorithm and (2) the Adaptive Recursive ARMA LMS algorithm. It was found that not only are the two LMS filtering techniques special suboptimal cases of the AKI; but, further, the AKI exhibits superior convergence and modeling properties for the cases where (1) the

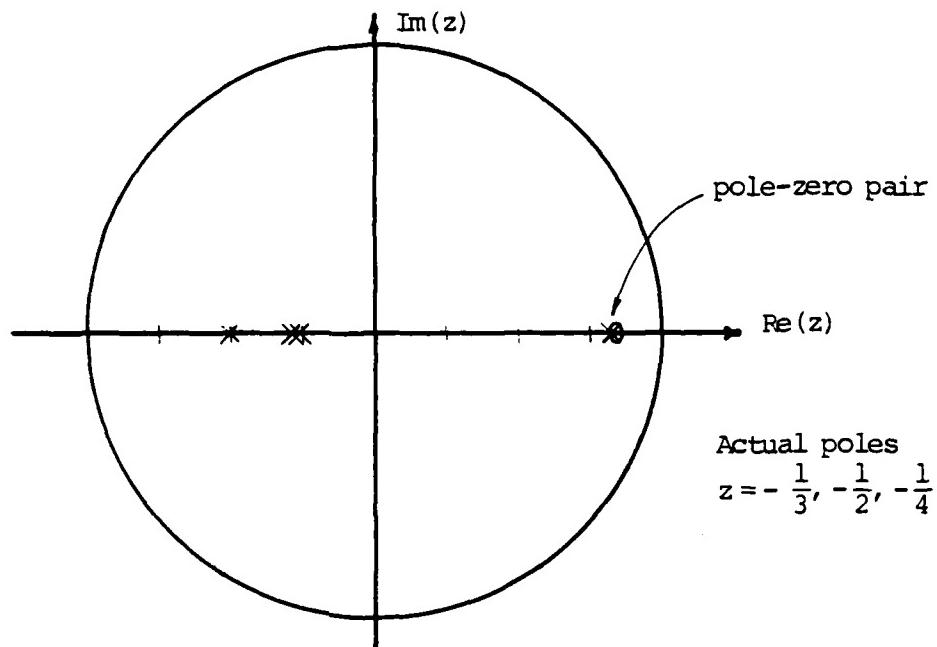


Figure 8.26. AKI(3,1) Overmodeled AKI(4,2)

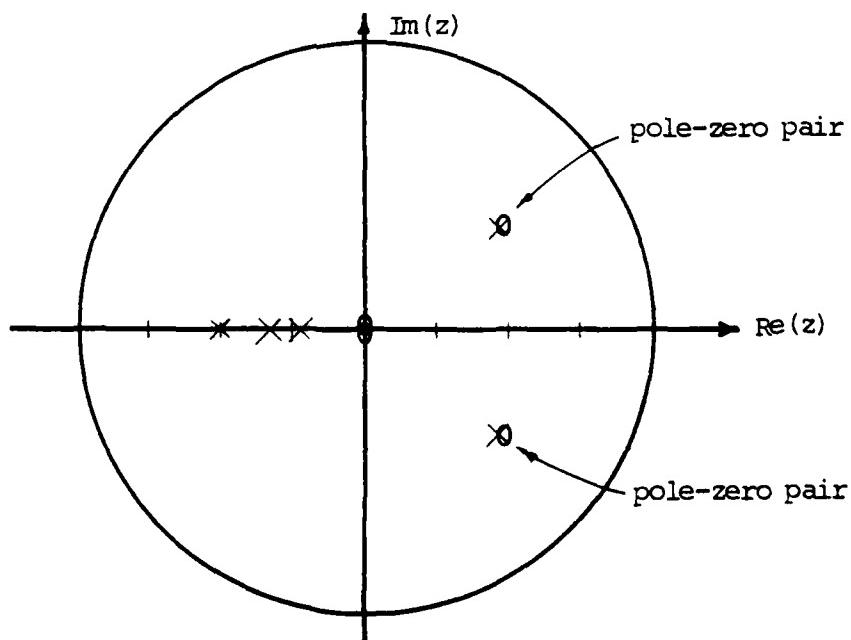


Figure 8.27. AKI(3,1) Overmodeled AKI(5,5)

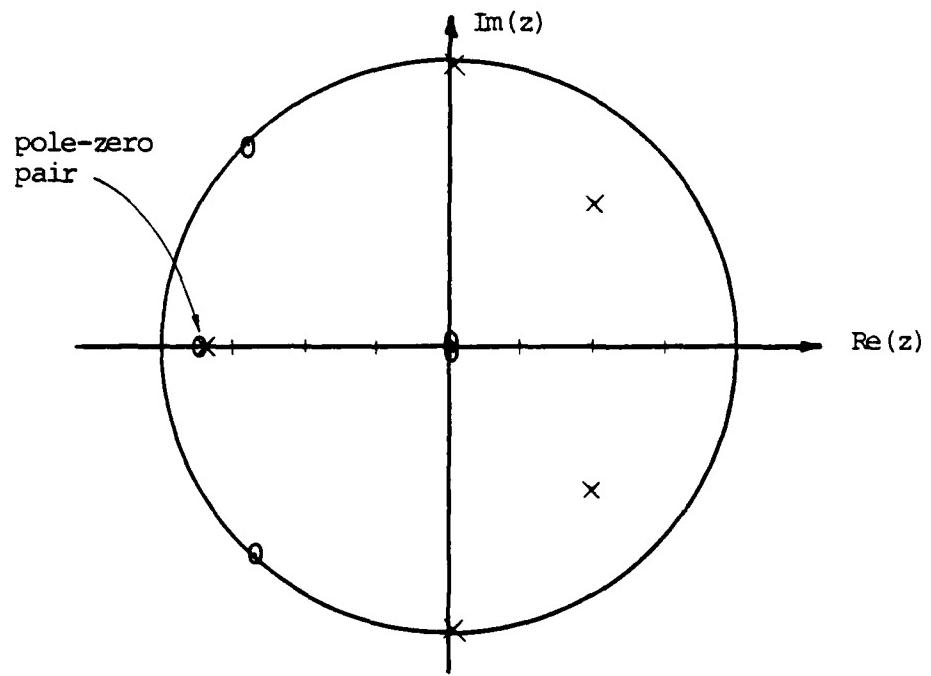


Figure 8.28. AKI(4,5) Overmodeled AKI(5,6)

order of the unknown plant is known and (2) the order of the plant is overmodeled. Additionally, the simulations indicate that accuracies similar to those obtained using lattice modeling methods, can be achieved using the AKI at a decidedly smaller number of iterations.

By making minor modifications to the measurement vector,  $H(k)$ , that is by using hybrid signals, the AKI was used to identify the linear and non-linear ARMA representations of a phase locked loop with success. Interestingly, the AKI technique appears to enable one to discern when a potential non-linear system enters its non-linear mode of operation, by closely monitoring the coefficients of the linear portion of the generalized non-linear ARMA model.

#### D. TOPICS FOR FURTHER CONSIDERATION

Several areas for further study directly and indirectly related to the AKI were uncovered. Foremost, a rigorous convergence proof is desirable. Although the connection was made between the AKI algorithm and the initial equation from which the lattice modeling algorithm is developed, similar comparisons (such as Chapters III and IV) could provide more insight into the operation of both. The multichannel AKI is the logical development of the single channel AKI presented. And, lastly, refinement of the AKI software (using the NL = 5 option) to include ARMA modeling of time series using a Box-Jenkins approach [Ref. 26] is feasible. A limited number of simulations using Monterey rain data and series C [Ref. 26]

data seem to indicate that an application exists for the AKI  
in this area.

APPENDIX A  
THE DISCRETE WEINER PROBLEM

The discrete Weiner problem considers optimally filtering a desired signal from unwanted stationary noise. The criterion of optimality used is minimization of the mean squared error between the output and the desired signal. Generally one desires that the optimal filter, which is the device being sought, be time invariant and that it be able to accept a signal,  $s(k)$ , and noise,  $n(k)$ , where each are samples from stationary random processes. In other words, we want a device which can accept

$$x(k) = s(k) + n(k) \quad (A.1)$$

as an input and produce at its output  $s(k+\Delta)$  or some linear function thereof where  $\Delta$  is some known delay.

Assuming that the desired output,  $d(k)$ , is the response to a specified sampled data linear system whose transfer function,  $H_d(z)$ , is given, then the error between the desired output and the output of the filter,  $y(k)$ , we seek is,

$$e(k) = d(k) - y(k). \quad (A.2)$$

Figure A.1 depicts the discrete Weiner problem formulation. The derivation from here follows the one presented by Maybeck [Ref. 27] for the continuous case. From the block diagram we have,

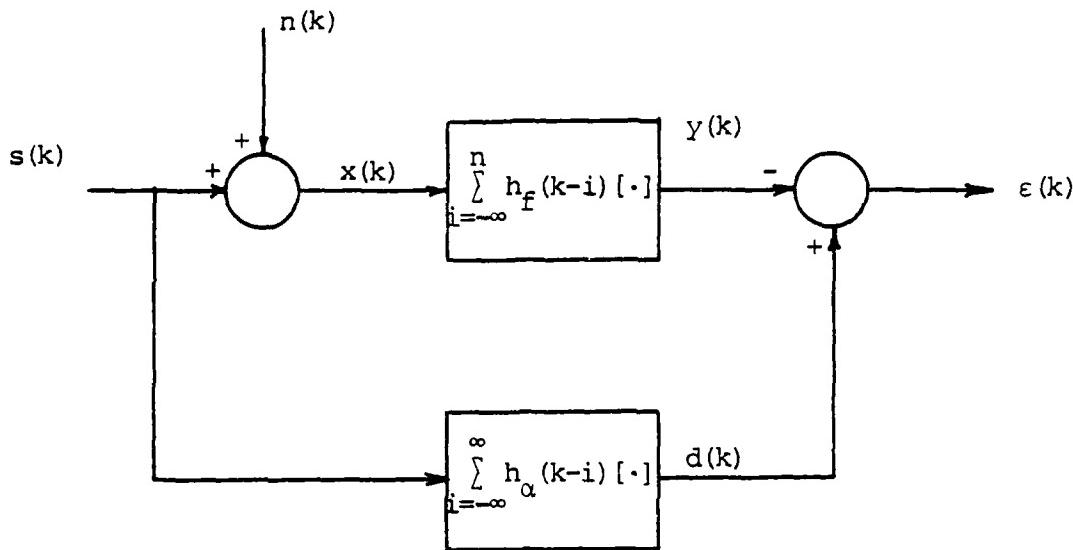


Figure A.1. The Discrete Weiner Problem

$$y(k) = \sum_{i=-\infty}^{\infty} h_f(k-i) x(i) \quad (A.3)$$

or equivalently,

$$y(k) = \sum_{i=-\infty}^{\infty} x(k-i) h_f(i) \quad (A.4)$$

Substituting equation A.4 into A.2 and squaring we have,

$$\begin{aligned} e^2(k) &= d^2(k) - 2d(k) \sum_{i=-\infty}^{\infty} h_f(i)x(k-i) \\ &\quad + [\sum_{i=-\infty}^{\infty} x(i)h_f(k-i)][\sum_{i=-\infty}^{\infty} x(i)h_f(k-i)] \end{aligned} \quad (A.5)$$

Taking the expected value of A.5 we obtain,

$$\begin{aligned} E\{e^2(k)\} &= \psi_{dd}(0) - 2 \sum_{i=-\infty}^{\infty} h_f(i) \psi_{dx}(i) \\ &\quad + \sum_{i=-\infty}^{\infty} h_f(i) \sum_{j=-\infty}^{\infty} h_f(j) \psi_{xx}(j-i) \end{aligned} \quad (A.6)$$

where notation  $\psi_{uv}(m)$  denotes the expected value of the product of  $u(k)$  and  $v(k+m)$ . That is,

$$\psi_{uv}(m) = E\{u(k)v(k+m)\} = E\{u(k)v(k-m)\} \quad (A.7)$$

otherwise known as the autocorrelation of  $u(k)$  and  $v(k)$ .

Using variational techniques and letting,

$$h_f(i) = h_{opt}(i) + \epsilon \Delta h(i) \quad (A.8)$$

substituting equation A.8 into equation A.6, taking the partial derivative with respect to epsilon,  $\epsilon$ , and setting the partial derivative equal to zero we have:

$$\sum_{n=-\infty}^{\infty} \Delta h(n) \left[ \sum_{i=-\infty}^{\infty} h_{opt}(i) \psi_{xx}(n-i) - \psi_{xd}(n) \right] = 0 \quad (A.9)$$

The term within the brackets is the discrete form of the Weiner-Hopf equation and must be equal to zero if equation A.9 is to be valid since  $\Delta h(n) > 0$  by definition. Therefore,

$$\sum_{i=-\infty}^{\infty} h_{opt}(i) \psi_{xx}(n-i) = \psi_{xd}(n) \quad (A.10)$$

Equation A.10 is the most often encountered in truncated form in linear prediction theory, Pade approximation, adaptive filtering and lattice filtering. The truncated version of A.10 which is generally used in solving the discrete Weiner problem is,

$$\sum_{i=0}^M h_{\text{opt}}(i) \psi_{xx}(n-i) = \psi_{xd}(n) \quad (\text{A.11})$$

or in matrix form,

$$\begin{bmatrix} \psi_{xx}(0) & \psi_{xx}(-1) & \dots & \psi_{xx}(-n) \\ \psi_{xx}(1) & \psi_{xx}(0) & \dots & \psi_{xx}(1-n) \\ \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \psi_{xx}(n) & \psi_{xx}(n-1) & \dots & \psi_{xx}(0) \end{bmatrix} \begin{bmatrix} h_{\text{opt}}(0) \\ h_{\text{opt}}(1) \\ \vdots \\ \vdots \\ h_{\text{opt}}(n) \end{bmatrix} = \begin{bmatrix} \psi_{xd}(0) \\ \psi_{xd}(1) \\ \vdots \\ \vdots \\ \psi_{xd}(n) \end{bmatrix} \quad (\text{A.12})$$

## APPENDIX B

### PROGRAM AND AVERAGE MOVING AVERAGE SIMULATION

THIS PROGRAM GENERATES THE VARIOUS TYPES OF LINEAR SYSTEMS DATA FOR THE ADAPTIVE KALMAN IDENTIFIERS (AKI). THE PROGRAM PERFORMS ALL COMPUTATIONS IN DOUBLE PRECISION AND SIGNS THE OUTPUT DATA ON A CDS FILE. THIS FILE IS ANALYZED BY THE AKI. THE AUTOREGRESSIVE (AR) AND MOVING-AVERAGE (MA) PROCESSES ARE PREDICTED AND DATA IS UNPACKAGED FOR THE AKI.

ROUTINE V(100),V(800),Z(600),V(3000)  
RT AL\*+ W(800),W(800)

ROUTINE C(100),X(100),A(100),B(100),C(100),D(100),  
DOLV = 1.00E-3  
SDEV = 1.00E-1

NP = NUMBER OF DATA POINTS FOR INPUT  
N = NUMBER OF ADAPTIVE COEFFICIENTS  
M = NUMBER OF MA COEFFICIENTS  
NL = NUMBER OF OPERATIONS FOR THE AKI  
NPER = INITIAL ORDER OF SYSTEM  
N=2  
M=6  
NL=2  
NPER=3

WRITE(\*,\*) N,P,NL

CALL DOLV(DOLV,NP,NL)  
CALL DPER(OP(NL),NP,NL)

DO I=1,LP  
V(I)=SIG(V(1),W(2,I))  
LO U(I)=DET(G(I))

A1=1.000  
A2=0.333  
A3=0.250  
A4=0.200  
A5=1.000  
A6=0.333  
A7=0.250  
A8=0.200

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A<sub>1</sub><sub>2</sub><sub>3</sub><sub>4</sub><sub>5</sub><sub>6</sub><sub>7</sub><sub>8</sub>

SOL 101 INITIAT CONDITIONS

$$\begin{aligned} u_1 &= 1.0 \\ u_2 &= 0.0 \\ u_3 &= 0.0 \\ u_4 &= 0.0 \\ u_5 &= 0.0 \\ u_6 &= 0.0 \\ u_7 &= 0.0 \\ u_8 &= 0.0 \end{aligned}$$

NR=0

$$\begin{aligned} u_1 &= 0 \\ u_2 &= 0 \\ u_3 &= 0 \\ u_4 &= 0 \\ u_5 &= 0 \\ u_6 &= 0 \\ u_7 &= 0 \\ u_8 &= 0 \end{aligned}$$
$$Y_{(n)} = A_{1-4} Y_{(K-1)} + A_{2-4} Y_{(K-2)} + A_{3-4} Y_{(K-3)} + A_{4-4} Y_{(K-4)} + A_{5-4} U_{(K-4)}$$
$$A_{1-4} = \frac{1}{(K-1)} + \frac{1}{(K-2)} + \frac{1}{(K-3)} + \frac{1}{(K-4)}$$
$$A_{2-4} = \frac{1}{(K-2)} + \frac{1}{(K-3)} + \frac{1}{(K-4)}$$
$$A_{3-4} = \frac{1}{(K-3)} + \frac{1}{(K-4)}$$
$$A_{4-4} = \frac{1}{(K-4)}$$

KK = KK+1

$$u_1 = W_{11} U_{(K)} + W_{12} U_{(K-1)} + W_{13} U_{(K-2)} + W_{14} U_{(K-3)}$$

$$u_2 = W_{21} U_{(K)} + W_{22} U_{(K-1)} + W_{23} U_{(K-2)}$$

$$u_3 = W_{31} U_{(K)} + W_{32} U_{(K-1)}$$

$$u_4 = W_{41} U_{(K)} + W_{42} U_{(K-1)}$$

SLP

LL

## APPENDIX C

### POSTCARDS FOR: FIRST TICKET LINE (PLL)

THIS PRACTICALLY ROADS THE MIGRANT FARM MIGRANT FARM PRODUCTS IN THE  
BETTER LIFES REALIZED BY THE MIGRANT FARMERS (S&K).  
THE SICKENED FARMERS ARE PHYSICALLY AND MENTALLY  
WITH THE WITH BROKEN MIGRANT FARMERS SINCE  
COSTLY BUT THE MIGRANT IS PAUPERILY SUPPORTED BY  
THE LOCALITIES:

$$P_{\alpha\beta}(L^{*\alpha}-1) = \frac{1 + AL^{\alpha}L^{*\alpha}-1 + AL^{\alpha}L^{*\alpha}-2}{1 - AL^{\alpha}L^{*\alpha}-1 - AL^{\alpha}L^{*\alpha}-2} \dots$$

$$\begin{aligned} & \text{AL}^{\alpha}L^{*\alpha}-1, L(12), 1, \lambda(1200), \gamma(1200), L(2000), \\ & L(2000), \lambda(2000), \gamma(2000), \lambda(2000), \gamma(2000), \lambda(2000), \gamma(2000) \\ & \text{AL}^{\alpha}L^{*\alpha}-2, L(12), 2000, \lambda(2000), \gamma(2000), L(2000) \end{aligned}$$

INITIAL TICKET LINE

$\frac{d^2x}{dt^2}$ ,  
 $\frac{dx}{dt}$

$$\begin{aligned} & \text{AL}^{\alpha}L^{*\alpha}-1, L(12), 1, \lambda(1200), \gamma(1200), L(2000), \gamma(2000) \\ & L(2000), \lambda(2000), \gamma(2000), \lambda(2000), \gamma(2000), L(2000), \gamma(2000) \end{aligned}$$

$$\begin{aligned} & \text{AL}^{\alpha}L^{*\alpha}-2, L(12), 2000, \lambda(2000), \gamma(2000), L(2000), \gamma(2000) \\ & L(2000), \lambda(2000), \gamma(2000), \lambda(2000), \gamma(2000), L(2000), \gamma(2000) \end{aligned}$$

$$\begin{aligned} & \text{AL}^{\alpha}L^{*\alpha}-3, L(12), 3000, \lambda(3000), \gamma(3000), L(3000), \gamma(3000) \\ & L(3000), \lambda(3000), \gamma(3000), \lambda(3000), \gamma(3000), L(3000), \gamma(3000) \end{aligned}$$

$$\begin{aligned} & \text{AL}^{\alpha}L^{*\alpha}-4, L(12), 4000, \lambda(4000), \gamma(4000), L(4000), \gamma(4000) \\ & L(4000), \lambda(4000), \gamma(4000), \lambda(4000), \gamma(4000), L(4000), \gamma(4000) \end{aligned}$$

$$\begin{aligned} & \text{AL}^{\alpha}L^{*\alpha}-5, L(12), 5000, \lambda(5000), \gamma(5000), L(5000), \gamma(5000) \\ & L(5000), \lambda(5000), \gamma(5000), \lambda(5000), \gamma(5000), L(5000), \gamma(5000) \end{aligned}$$

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DRUGS FOR PARKINSON'S DISEASE

CONVENTIONAL IMPORTS AND SET THE FUTURE LEVEL  
 $\frac{\partial \pi_{\text{con}}}{\partial P} = \frac{\frac{\partial \pi_{\text{con}}}{\partial P} - \frac{\partial \pi_{\text{con}}}{\partial P}}{\left( \frac{\partial \pi_{\text{con}}}{\partial P} + \frac{\partial \pi_{\text{con}}}{\partial P} \right)} = \frac{1.50 - 0.60}{1.50 + 0.60} = 0.40$

WITH A SAMPLE SIZE OF 100, THE ESTIMATE IS 1.000, WHICH IS A CALORIAC INPUT OF 100 KJ.

DR. H. SAWYER PLEASE LOOKED UP  
TODAY.

Reinstated to service after simulation

COLLECTED DATA IN FILTERS OF THE FIBRILLAR ANALYSIS  
 $\alpha_1 = \frac{\alpha_1}{\alpha_1 + \alpha_2}$   
 $\alpha_2 = \frac{\alpha_2}{\alpha_1 + \alpha_2} = \text{SOL}(\text{V}(1)) + \text{SOL}(\text{V}(2))$   
 $\alpha_3 = \frac{\alpha_3}{\alpha_1 + \alpha_2 + \alpha_3} = \text{SOL}(\text{V}(3))$   
 $\alpha_4 = \frac{\alpha_4}{\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4} = \text{SOL}(\text{V}(4))$

Fig. 10.1. CDF of the empirical CATA based on 5000 simulated samples of size 100. The solid line is the theoretical CDF of the CATA distribution. The dashed line is the empirical CDF.

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DO... COTTON 0.800, (PC, C, D, M, A, Y, MP, LC)

SUBJ: 0.800 0.800 INVESTIGATE PLANT 11 SISTER  
TO OUTLET OF TATTE MARKET = 504 AT THE HUT CHANAL.  
PLTS ARE CROWN CHANNEL.

1. IN 0.800 0.800, RP 1(1,1)  
RP 2(1,1), Y(1,1) X(1,1), Y(1,1), X(1,1)  
CROWN (1,1), Y(1,1) Y(1,1) Y(1,1)  
CROWN (1,1), Y(1,1)

IN 0.800 0.800 STAIN FRESH COT COT COT  
LHAC-6

2. IN 0.800 0.800 TATTE PL  
RHAC-6

3. IN 0.800 0.800 GALT PL 0.8  
GALT (1,1) X(1,1) Y(1,1) P(1,1)  
L(1,1) Y(1,1) X(1,1) P(1,1) Y(1,1) P(1,1)  
L(1,1) Y(1,1) X(1,1) P(1,1) Y(1,1) P(1,1)  
L(1,1) Y(1,1) X(1,1) P(1,1) Y(1,1) P(1,1)

4. IN 0.800 0.800 TATTE Y(1,1)  
Y(1,1) - COT.

5. IN 0.800 0.800 TATTE Y(1,1) COT  
TATTE Y(1,1) COT  
SAY 0.800 0.800 TATTE Y(1,1)  
SAY 0.800 0.800 TATTE Y(1,1)  
SAY 0.800 0.800 TATTE Y(1,1)

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6. IN 0.800 0.800 TATTE Y(1,1)  
TATTE Y(1,1) TATTE Y(1,1) TATTE Y(1,1)  
TATTE Y(1,1) TATTE Y(1,1) TATTE Y(1,1)  
TATTE Y(1,1) TATTE Y(1,1) TATTE Y(1,1)





## APPENDIX D

CARBONATE ROCKS AND SEDIMENTATION IN THE TURKANA BASIN

PROGRAM ALARM: ADAPTIVE FACTOR: LOAD LIMIT (AKI)

THIS PROGRAM PERIODICALLY CHECKS THE MAIN SYSTEM WHICH IS ADAPTIVE TO THE KALMAN FILTER. THE CAPTURED DATA IS USED TO COMPUTE THE POSITION OF THE CENTER OF GRAVITY OF THE UNKNOWN UNKNOWN SYSTEM. THEREFORE, THIS SYSTEM IS SIMPLY AUGMENTED WITH THE STATE VECTOR OF THE UNKNOWN SYSTEM. USING THE ADAPTIVE DOCUMENTATION OF THE SYSTEM IS MADE. BUT, IN ORDER TO DO SUCH A DOCUMENTATION, THE SYSTEM MUST BE IDENTIFIED PRECISELY. THE PAPER LISTS THE ABOVE IDENTIFICATION ARE IDENTIFIED BY IDENTIFYING THE AKI EFFICIENTLY. THE AKI EFFICIENTLY STORED ON A DISK FILE PERTAINING THE ADJUSTMENT SEQUENCES OF THE OPTIMUM FLAG. THE OPTIMUM FLAG IS USED IN THE CREATION OF THE AKI PROGRAM WHICH IS DESCRIPTED THROUGHOUT THE DOCUMENTATION.

1. [www.123rf.com](http://www.123rf.com) 2. [www.123rf.com](http://www.123rf.com) 3. [www.123rf.com](http://www.123rf.com) 4. [www.123rf.com](http://www.123rf.com) 5. [www.123rf.com](http://www.123rf.com) 6. [www.123rf.com](http://www.123rf.com) 7. [www.123rf.com](http://www.123rf.com) 8. [www.123rf.com](http://www.123rf.com) 9. [www.123rf.com](http://www.123rf.com) 10. [www.123rf.com](http://www.123rf.com)

RESULTS OF THE CALCULATIONS ARE AS FOLLOWS:

THE JOURNAL OF CLIMATE

2007-08 NATIONAL CERTIFICATION ACTIVITIES FOR THE  
NATIONAL PEDIATRIC CRITICAL CARE WORKER;  
2007-08 SYSTEMIC RURAL CRITICAL CARE FOR THE

卷之三

$$n=1$$

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SOCIOPOLITICAL S

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$$\frac{d}{dt} \left( \frac{d}{dt} \right) = \frac{d}{dt} \left( \frac{d}{dt} \right) + \frac{d}{dt} \left( \frac{d}{dt} \right) = \frac{d}{dt} \left( \frac{d}{dt} \right) + \frac{d}{dt} \left( \frac{d}{dt} \right)$$

1. *U.S. Fish Commission*, *Report for the Year 1873* (Washington, 1875), p. 11.

REVIEW OF THE LITERATURE ON UNKNOWN PLANT LIFE

IN THE MOUNTAINS OF ALABAMA AND IN YUKON AND

THE BOSTONIAN

卷之三

THE RENAISSANCE OF THE MUSICAL DRAMA

1. *W* is a  $\mathbb{C}$ -vector space, and  $\mathcal{A} = \text{End}_{\mathbb{C}}(W)$  is the algebra of linear operators on  $W$ .  
 2.  $\mathcal{A}$  is a  $\mathbb{C}$ -algebra.  
 3.  $\mathcal{A}$  is a  $\mathbb{C}$ -algebra with unit element  $1_W$ .

卷之三

$$\frac{1}{2} \left( \frac{\partial^2 \phi}{\partial x^2} - \frac{\partial^2 \phi}{\partial y^2} \right) = \frac{1}{2} \left( \frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^2 \psi}{\partial y^2} \right)$$

CARTOGRAFIA DE LA DISTRIBUCIÓN DE LOS PECES EN EL RÍO TAMBORÉ

卷之三

卷之三十一

TAKI, TAKI ET AL. / MIGRATION AND HABITAT USE 11

$$e_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad e_3 = \begin{pmatrix} N & 0 \\ 0 & 1 \end{pmatrix}, \quad e_4 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

THEORETICAL

1976年1月1日～1977年1月1日の間に、日本全国で発生した火災の統計

卷之三

卷之三

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LOCATE INITIAL VALUES OF THE PASTORAL VECTORS

IN  $(K+1)$  AND  $K$ .

FOR FIRST PASTORAL VECTOR WITH  $Y(K+1)$  ELEMENTS

$L_1 = L_{K+1}$

$Z(C, R(1,1)) = Y(1)(L_1 - K + 1)$

40 Construct

$R(R, C, 0) = R(0, C, 0)$

LOCATE PASTORAL VECTOR WITH  $Y(K+1)$  ELEMENTS

$L_1 = L_{K+1}$

$Z(C, R(1,1)) = Y(1)(L_1 - K + 1)$

41 Compute  $R(R, C, 0) \cdot (R(1,1) \cdot Z(C, R(1,1)))$  IN  $R(R, C, 0)$

FOR EACH SUBVECTOR WITH NON-LINEAR ELEMENTS  $Y(K+1) \cdot Y(K)$

$R(R, C, 0) = R(R, C, 0) + R(R, C, 0) \cdot Z(C, R(1,1))$

Locate first element of vector with nonlinear elements  $Y(K+1) \cdot Y(K)$

$L_1 = L_{K+1}$

$Z(C, R(1,1)) = Y(1)(L_1 - K + 1)$

FOR EACH SUBVECTOR WITH NON-LINEAR ELEMENTS  $Y(K+1) \cdot Y(K)$

$L_1 = L_{K+1}$

$Z(C, R(1,1)) = Y(1)(L_1 - K + 1)$

42 Compute  $R(R, C, 0) \cdot (R(1,1) \cdot Z(C, R(1,1)))$  IN  $R(R, C, 0)$

FOR EACH SUBVECTOR WITH NON-LINEAR ELEMENTS  $Y(K+1) \cdot Y(K)$

$L_1 = L_{K+1}$

$Z(C, R(1,1)) = Y(1)(L_1 - K + 1)$

43 Compute  $R(R, C, 0) \cdot (R(1,1) \cdot Z(C, R(1,1)))$  IN  $R(R, C, 0)$

$L_1 = L_{K+1}$

$Z(C, R(1,1)) = Y(1)(L_1 - K + 1)$

44 Compute  $R(R, C, 0) \cdot (R(1,1) \cdot Z(C, R(1,1)))$  IN  $R(R, C, 0)$

$L_1 = L_{K+1}$

$Z(C, R(1,1)) = Y(1)(L_1 - K + 1)$

45 Compute  $R(R, C, 0) \cdot (R(1,1) \cdot Z(C, R(1,1)))$  IN  $R(R, C, 0)$

$L_1 = L_{K+1}$

$Z(C, R(1,1)) = Y(1)(L_1 - K + 1)$





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For each  $\alpha \in \{0, 1, 2, 3\}$ ,  $P_{\alpha}(\theta) = \frac{1}{4} \left( \sum_{i=0}^3 P_i(\theta) \right)$ . The probability distributions  $P_0(\theta), P_1(\theta), P_2(\theta), P_3(\theta)$  are given by the following tables:

ESTATE PLANNING AND INVESTMENT PORTFOLIO MANAGEMENT

卷之三十一

0 (0.000000) at 10.000  
0.000000 (0.000000)  
1 (0.000000) at 10.000  
0.000000 (0.000000)  
2 (0.000000) at 10.000  
0.000000 (0.000000)  
3 (0.000000) at 10.000  
0.000000 (0.000000)

卷之三

1.  $\left( \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right)$   $\rightarrow$   $\left( \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right)$   
 $\left( \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right)$   $\rightarrow$   $\left( \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right)$   
 $\left( \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right)$   $\rightarrow$   $\left( \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right)$   
 $\left( \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right)$   $\rightarrow$   $\left( \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right)$   
 $\left( \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right)$   $\rightarrow$   $\left( \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right)$

卷之三

For  $\alpha = 1$ ,  $\beta = 0$ ,  $\gamma = 0$ ,  $(1, 1)$  is a local minimum.

$$P_{k+1}(x/k+1) = \frac{1}{G(1, k+1)} \cdot \frac{1}{G(1, k+2)} \cdot \frac{1}{G(1, k+3)} \cdot \dots \cdot \frac{1}{G(1, k+1+k-1)} = \frac{1}{G(1, k+1)} \cdot \frac{1}{G(1, k+2)} \cdot \frac{1}{G(1, k+3)} \cdot \dots \cdot \frac{1}{G(1, k+1+k-1)}$$

1.0.  $P_{k+1}(x/k+1) = \frac{1}{G(1, k+1)} \cdot \frac{1}{G(1, k+2)} \cdot \frac{1}{G(1, k+3)} \cdot \dots \cdot \frac{1}{G(1, k+1+k-1)}$

1.1.  $x/k+1 = \frac{x}{k+1} + \frac{k}{k+1}$  Then we have  $x/k+1 = x/k + 1$

1.1.4.  $P_{k+1}(x/k+1) = P_k(x/k)$

1.1.5.  $P_{k+1}(x/k+1) = P_k(x/k)$

1.1.6.  $P_{k+1}(x/k+1) = (1 - e^{-\lambda_k}) P_k(x/k) + e^{\lambda_k} P_k(x/k+1)$

1.1.7.  $P_{k+1}(x/k+1) = (1 - e^{-\lambda_k}) P_k(x/k) + e^{\lambda_k} P_k(x/k+1)$

1.1.8.  $P_{k+1}(x/k+1) = (1 - e^{-\lambda_k}) P_k(x/k) + e^{\lambda_k} P_k(x/k+1)$   
 Total price  $(1 - e^{-\lambda_k}) P_k(x/k) + e^{\lambda_k} P_k(x/k+1)$  is  $P_k(x/k)$

1.1.9.  $P_{k+1}(x/k+1) = P_k(x/k)$

1.1.10.  $P_{k+1}(x/k+1) = P_k(x/k) + e^{-\lambda_k} P_k(x/k+1)$

1.1.11.  $P_{k+1}(x/k+1) = P_k(x/k) + e^{-\lambda_k} P_k(x/k+1)$

1.1.12.  $P_{k+1}(x/k+1) = P_k(x/k) + e^{-\lambda_k} P_k(x/k+1)$

1.1.13.  $P_{k+1}(x/k+1) = P_k(x/k) + e^{-\lambda_k} P_k(x/k+1)$

1.1.14.  $P_{k+1}(x/k+1) = P_k(x/k) + e^{-\lambda_k} P_k(x/k+1)$

1.1.15.  $P_{k+1}(x/k+1) = P_k(x/k) + e^{-\lambda_k} P_k(x/k+1)$

1.1.16.  $P_{k+1}(x/k+1) = P_k(x/k) + e^{-\lambda_k} P_k(x/k+1)$

1.1.17.  $P_{k+1}(x/k+1) = P_k(x/k) + e^{-\lambda_k} P_k(x/k+1)$

1.1.18.  $P_{k+1}(x/k+1) = P_k(x/k) + e^{-\lambda_k} P_k(x/k+1)$

1.1.19.  $P_{k+1}(x/k+1) = P_k(x/k) + e^{-\lambda_k} P_k(x/k+1)$

1.1.20.  $P_{k+1}(x/k+1) = P_k(x/k) + e^{-\lambda_k} P_k(x/k+1)$

1.1.21.  $P_{k+1}(x/k+1) = P_k(x/k) + e^{-\lambda_k} P_k(x/k+1)$

1.1.22.  $P_{k+1}(x/k+1) = P_k(x/k) + e^{-\lambda_k} P_k(x/k+1)$

1.1.23.  $P_{k+1}(x/k+1) = P_k(x/k) + e^{-\lambda_k} P_k(x/k+1)$

1.1.24.  $P_{k+1}(x/k+1) = P_k(x/k) + e^{-\lambda_k} P_k(x/k+1)$

1.1.25.  $P_{k+1}(x/k+1) = P_k(x/k) + e^{-\lambda_k} P_k(x/k+1)$

1.1.26.  $P_{k+1}(x/k+1) = P_k(x/k) + e^{-\lambda_k} P_k(x/k+1)$

1.1.27.  $P_{k+1}(x/k+1) = P_k(x/k) + e^{-\lambda_k} P_k(x/k+1)$

1.1.28.  $P_{k+1}(x/k+1) = P_k(x/k) + e^{-\lambda_k} P_k(x/k+1)$

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ALG 54112

ALG 54113

ALG 54114

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ALG 54116

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ALG 54119

ALG 54120

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ALG 54140

ALG 54141

ALG 54142

ALG 54143

ALG 54144

ALG 54145

ALG 54146

ALG 54147

ALG 54148

Color CRT:  $\text{shk}(\text{kr}, \text{ks}+1) = \text{shk}(\text{kr}) * \text{shk}(\text{ks})$

32. *GRADIENT*

33.  $\frac{\partial}{\partial x} \text{values at } (k+1) = \text{Ainst}(1,k) + \epsilon(1,F+1) * t.b$   
34.  $\frac{\partial}{\partial x}$   
35.  $\frac{\partial}{\partial y}$

36.  $\text{SOLUTIVE EQUATION } (\text{A}, N, P, Q)$   
37.  $\text{THIS SOURCE IS FOR THE PRACTICAL USE OF THE STIRLING'S FORMULA}$

38.  $\frac{\partial^2}{\partial x^2} \text{values at } (1,k+1) = (0,0,20)$

39.  $\frac{\partial^2}{\partial x^2} \text{values at } (1,k+1) = \frac{1}{2}(1,1,1)$

40.  $\frac{\partial^2}{\partial y^2}$

41.  $\text{SOLUTIVE EQUATION } (\text{A}, N, P, Q)$   
42.  $\text{THIS SOURCE IS FOR THE PRACTICAL USE OF THE STIRLING'S FORMULA}$   
43.  $\frac{\partial^2}{\partial x^2} \text{values at } (1,k+1) = (0,0,20), (0,0,20), (0,0,20)$

44.  $\frac{\partial^2}{\partial x^2} \text{values at } (1,k+1) = (0,0,20)$   
45.  $\frac{\partial^2}{\partial x^2} \text{values at } (1,k+1) = (0,0,20)$   
46.  $\frac{\partial^2}{\partial x^2} \text{values at } (1,k+1) = (0,0,20)$   
47.  $\frac{\partial^2}{\partial y^2} \text{values at } (1,k+1) = (0,0,20)$

obtained by the following steps:

1. Let  $\mathcal{N}$  be the set of nodes in  $G$  and  $M$  be the set of edges in  $G$ .

$$\begin{aligned} & \text{Let } A = \{e \in M : e \text{ is a bridge}\} \\ & \text{Let } B = \{e \in M : e \text{ is not a bridge}\} \end{aligned}$$

2. Let  $\mathcal{N}_A$  be

the set of nodes  $v$  such that  $(v, v)$  is a bridge edge in  $G$ .  
Let  $\mathcal{N}_B$  be the set of nodes  $v$  such that  $(v, v)$  is not a bridge edge in  $G$ .

3. Let  $\mathcal{E}_A$  be the set of edges  $e$  such that  $e$  is a bridge edge in  $G$ .  
Let  $\mathcal{E}_B$  be the set of edges  $e$  such that  $e$  is not a bridge edge in  $G$ .

$$\begin{aligned} & \text{Let } C = \{e \in M : e \text{ is a cycle edge}\} \\ & \text{Let } D = \{e \in M : e \text{ is not a cycle edge}\} \end{aligned}$$

4. Let  $\mathcal{N}_C$  be

the set of nodes

This function takes a vector  $\mathbf{v}$  and a matrix  $M$  as input and returns the result of the multiplication  $M \cdot \mathbf{v}$ . It uses a loop to iterate through each row of  $M$  and multiply it by the corresponding element of  $\mathbf{v}$ . The result is stored in a temporary variable  $temp$  and then added to the current value of  $result$ . Finally, the function returns the total result.

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SOON TO FOLLOW. (144)  
AND, CONSIDERED, AND, CONSIDERED, IN THE MEANTIME, AT THE PRESENT  
TIME, THE, (144) . . . YESTERDAY, (144) . . . (144) . . .

TOOK SO MUCH TIME, WORKED SO HARD, (144) . . . (144) . . .  
CONSIDERED, AND, CONSIDERED, AND, CONSIDERED, (144) . . . (144) . . .  
RECORDED, (144) . . . (144) . . . (144) . . . (144) . . . (144) . . .  
CONSIDERED, AND, CONSIDERED, AND, CONSIDERED, (144) . . . (144) . . .

SOON TO FOLLOW.  
IN THE MEANTIME, (144)  
IN THE MEANTIME, (144)  
IN THE MEANTIME, (144)

SOON TO FOLLOW.  
IN THE MEANTIME, (144)

SOON TO FOLLOW.

SOON TO FOLLOW.  
IN THE MEANTIME, (144)  
IN THE MEANTIME, (144)  
IN THE MEANTIME, (144)

SOON TO FOLLOW.  
IN THE MEANTIME, (144)  
IN THE MEANTIME, (144)  
IN THE MEANTIME, (144)

SOON TO FOLLOW.  
IN THE MEANTIME, (144)

SOON TO FOLLOW.  
IN THE MEANTIME, (144)  
IN THE MEANTIME, (144)

SOON TO FOLLOW.

SOON TO FOLLOW.  
IN THE MEANTIME, (144)

THE VARIOUS FORMS OF THE  
POLYMERIC POLY(1,4-PHENYLIC  
ACRYLIC ACID).

卷之三

Fig. 11. (Continued) (a) 42

卷之三

$$\frac{d}{dt} \left( \frac{1-t}{t} \right) = \frac{1}{t^2} \left( 1 - (1-t)(-1) \right) = \frac{1}{t^2} (1+1) = \frac{2}{t^2}$$

Journal of Health Politics, Policy and Law, Vol. 31, No. 1, January 2006

$$\begin{cases} \frac{1}{2} < \alpha < \frac{1}{2} + \frac{1}{n} \\ \frac{1}{2} - \frac{1}{n} < \beta < \frac{1}{2} \\ \frac{1}{2} - \frac{1}{n} < \gamma < \frac{1}{2} + \frac{1}{n} \end{cases}$$

卷之三

卷之三

REVIEWED BY DR. JAMES B. HARRIS, JR., DEPARTMENT OF POLITICAL SCIENCE, THE UNIVERSITY OF TORONTO

$\frac{1}{\lambda} \ln \frac{1}{P_{\text{err}}}$	$\frac{1}{\lambda} \ln \frac{1}{P_{\text{err}}}$	$\frac{1}{\lambda} \ln \frac{1}{P_{\text{err}}}$
1.0	1.0	1.0
1.5	1.5	1.5
2.0	2.0	2.0
2.5	2.5	2.5
3.0	3.0	3.0
3.5	3.5	3.5
4.0	4.0	4.0
4.5	4.5	4.5
5.0	5.0	5.0

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卷之三

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## APPENDIX E

```

PROGRAM 145: WIDROW LEAST MEAN SQUARES FILTER

THIS PROGRAM COMPUTES THE MOVING-AVERAGE(MA) LOOK EQUIVALENT LY
THE TRANSVERSAL FILTER WEIGHTS(COEFFICIENTS) OF A LEAST MEANS
SQUARES FILTER USING AN ADAPTIVE ALGORITHM WHICH IMPLEMENTS THE
METHOD OF STEEPEST DESCENT. THIS ALGORITHM WAS FIRST DEVISED
BY WIDROW.

LET #6 V(2000), X(12), Y(12), U(2000), F, KS, D, C
SET #4 V(12), X(12), Y(12), U(12,2000)
INTEGER M,NP,KK

THE CONVERGENCE FACTOR IS KS
KS = -2.0D-1

100 WRITE (6,100) KS
      PRINT#1,1,ICX,28;THE CONVERGENCE FACTOR: KS = ,F10.4,/
      INT LBL1 LR1,ICX,M
      INT LBL2 LR2,K1,4,K2
      READ (6) LR1,K1,4,K2
      WRITE (6,200) M
      FORMAT (1I8,15,2X,21WEIGHTS ARE ESTIMATED,/)
      INT DIAGNOSTIC FLAG IS IFLAG
      IFLAG = 0
      NP = 100
      WRITE (6,300)
      WRITE (6,31) 3X,5HERRCR,4X,6HOUTPUT,6X,4HW(1),6X,4HW(2)
      WRITE (6,32) 4HW(3),6X,4HW(4),6X,4HW(5),6X,4HW(6),6X,4HW(7),6X,4HW(8)
      WRITE (6,33) 4HW(9),5X,5HW(10),5X,5HW(11),5X,5HW(12),5X,5HW(13)
      INT NUMBER OF ITERATIONS IS NIP
      NIP = 100
      DO 10 I=1,M
      X(I) = 0.0D0
      Y(I) = 0.0D0
      U(I) = 0.0D0
      10 XX(I) = 0.0D0
      DO 20 KLBL1(6) 0 I=1,NP
      20 RLBL2(6) 0 I=1,NP
      U(I) = UFILE(V1(I+10))
      V(I) = UFILE(V2(I+10))

```

DO N=1,40 K=2, Np

$$U = Y(K-1)$$

DO I=1,10  
W(I)=((L1,A1,B1,I,0)\*U0+U1)  
W(I)=W(I)+D1\*SIN(C SIGNAL D(K):0,5X,E15.5,I)

END DO

COMPUTE T(N,K)  
IF T>E0 THEN GO TO 112

W1=IF (E2,120) (/,5X,\*W(I):,,10X,\*X(I):,,/,)

120 FOR I=1,3 DO W(I)=E140\*X(I)\*X(I)

130 FOR I=4,15 DO W(I)=2E15\*X(I)

140 FOR I=16,20 DO W(I)=2E15.4\*X(I)

CALL RMLL(W,X,A,C)  
END-C

W11=E((E4,90)\*K1)\*C, (W(I),I=1,N)

AL=k-1  
DO M=1,50 L=1,M  
M0(M,L)=SMGL(J(L))

150 FOR I=1,10 AL=E(G,0) GC TC 50

W1=IF (E2,50) (/,5X,\*X(I):,,10X,\*X(I):,,/,)

160 FOR I=1,6 DO W(I)=X(I),I=1,N

170 FOR I=7,15 DO W(I)=2E15\*X(I)

180 COMPUTE W(K+1): THE NEXT STEP PREDICTION

DO i=1,M  
W(i)=L1\*A(i)

A(X(i))=KS\*I\*X(I)

W(I)=W(I)-X(X(I))

SMGL I THE INPUT DATA VECTOR THRU THE DELAY LINE BY ONE  
L=L+1  
N=N-1  
DO I=1,N  
L1=L-1  
TO A(L1)=U(K)  
A(I)=U(L1-1)

4.0 CERN Input

6.0  $F = NP - 1$   
6.0 R11 E(4;160) R1,NP,P

16C PERAT (2;15)  
17C PERAT (4;130) MW(1,J)  
18C PERAT (4;20,5)

SUP  
HNU

LPS C1010  
LPS C1020  
LPS C1030  
LPS C1040  
LPS C1050  
LPS C1060  
LPS C1070  
LPS C1080  
LPS C1090  
LPS C1100  
LPS C1110  
LPS C1120  
LPS C1130  
LPS C1140

SUBROUTINE MULT(B,C,M,D)  
REAL A,B,C,D  
DIMENSION B(12),C(12)

INTEGER M

D=0.0  
DO 10 I=1,M  
D=D+B(I)\*C(I)  
10 RETURN

## **APPENDIX F**

TABLE I ASK: LEAST MEAN SQUARES RECURSIVE FILTER

TABLE II PREDICTION COMPUTES THE LMS ESTIMATES OF THE COEFFICIENTS OF THE LMS RECURSIVE FILTER BACKWARD AS WELL AS FEEDFORWARD LMS FILTERS.

THE BIGGEST FAULTY PROGRAM HAS THE FOLLOWING RESTRICTIONS:  
(1) The number of the main process must be greater than or equal  
to 2.  
(2) The number of the sub process must be greater than or equal to 1.

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THE CONVENTIONAL FACTORS ARE KA AND KB

WRITE ( $c_1, c_2$ ) KA, KB  
FUNCTION ( $c_1, c_2$ )  
CONVERGENCE FACTORS ARE :  $c_1, c_2$ , KA = 0,  
KB = 0  
IF ( $c_1 < 0$ ) THEN  
KA = 1  
IF ( $c_2 < 0$ ) THEN  
KB = 1  
END IF

THE FOLLOWING LETTER IS N  
K2 IS NOT USED IN THIS PROGRAM

2001 FOIST A 115X152X15 CURSIVE WEIGHTS ESTIMATED //

that *Lamonticula* is flag; the printing flag is *IPRI*

卷之三







C WRITE (6,201) (DB(I), I=1,N5)  
C 201 L=NPR1, NPP  
C 202 WRITE (6,203) DB(I,L), I=1,N5  
C 203 M TO 206  
C 205 WRITE (6,207)  
C 207 NUMERATOR IS INCORRECT; EITHER THE NUMERATOR OR THE DENOMINATOR IS INCORRECT; EITHER M OR N IS INCORRECTLY SPECIFIED. READ IN THE CORRECT ./5X VALUES FOR M AND N AND TRY AGAIN.  
C 206 CONTINUE  
C STOP  
C END

LPS C1760  
LPS C1970  
LPS C1980  
LPS C1990  
LPS 02000  
LPS 02010  
LPS 02020  
LPS C2020  
LPS C2030  
LPS C2040  
LPS C2050  
LPS C2060  
LPS C2070  
LPS C2080  
LPS 02090  
LPS C2100  
LPS C21100  
LPS C2120

SUBROUTINE WHDL (A,B,N,D)  
RET \* J E (12) \* E (12), J  
INT GR N  
L=0,000  
L=1,1=N  
D=L,A(1)\*D(1)  
RT JRN  
END

SC2150  
SC2160  
SC2170  
SC2180  
SC2190  
SC2200  
SC2210  
SC2220  
SC2230  
SC2240  
SC2250  
SC2260  
SC2270

SOURCELIST FILE (A,B,N,E)  
READ \*B, A(L2), B(L2), D  
IF,IT,GTR, N  
C=C, CLO  
DO,10,I=2,N  
10 D=L+A(I)+B(I)  
RETURN  
END

LPS02300  
LPS02310  
LPS02320  
LPS02330  
LPS02340  
LPS02350  
LPS02360  
LPS02370  
LPS02380  
LPS02390  
LPS02400

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